# Aggregate Implications of Heterogeneous Inflation Expectations: The Role of Individual Experience *

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**Abstract**

We show that inflation expectations are heterogeneous and depend on past individual experiences. We propose a history-based expectations-augmented Kalman filter to represent consumers' heterogeneous inflation expectations, where heterogeneity comes from an anchoring-to-the-past mechanism. Using survey data, we show that the model replicates US consumers' inflation expectations and its heterogeneity across cohorts. We introduce this mechanism into a New Keynesian model and find that heterogeneous expectations anchor aggregate responses to the agents' inflation history, producing sluggish expectations dynamics. Central banks should be active to prevent inflationary episodes that agents will remember far into the future.

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# I Introduction

Inflation expectations matter for decisions at both the firm and the household levels (Coibion et al., 2023, 2020; Hajdini et al., 2022b). Given their importance, there is an increased interest in measuring them and exploring what determines their formation process. The recent literature shows that individuals form inflation expectations, for instance, based on their recent buying experience (D'Acunto et al., 2021) and historical experiences regarding aggregate inflation (Malmendier & Nagel, 2016). While most studies on this topic provide empirical evidence regarding how differences in inflation expectations arise at the individual level and their effects on different micro-level decisions, there is less understanding of the aggregate implications associated with the inflation expectations' heterogeneity observed in the data. There is a noticeable gap in the literature between the empirical micro-level findings and macroeconomic models.

This paper aims to fill this gap. First, we show that individuals' inflation expectations depend on their history of inflation, confirming the main empirical finding of Malmendier & Nagel (2016) but using new US and international evidence.[1] Using detailed micro-level data, we find that (i) inflation expectations are heterogeneous across cohorts, (ii) inflation experiences are clustered by age, (iii) individual inflation history is positively correlated with inflation expectations, and (iv) there are no differences between cohorts in updating to current information, after controlling for their own history.

These facts suggest that consumers use their history of inflation to form expectations by positively weighing their past inflationary experiences.[2] We propose a model of inflation expectation-formation process where consumers weigh past experiences. We depart from the full information rational expectations model by presenting a framework where individuals use their past inflation histories and compare them to current information rationally gathered from signals.

Under the proposed framework, inflation expectations have two components: a forecast made

[1] While Malmendier & Nagel (2016) use the University of Michigan's Survey of Consumers (MSC), in sections III and IV of this paper, we provide external validity to their result by using a different data source with panel characteristics: the Survey of Consumer Expectations (SCE) from the Federal Reserve Bank of New York. In addition, in Appendix G, we show that the finding is consistent at the international level even after controlling for common cohort characteristics.

[2] This could interpreted as in the availability bias (Tversky & Kahneman, 1973), where consumers would use the information available (their experience) to form expectations, over-weighing their history instead of using more complex models that are necessary to form expectations about future economic variables.

with current shared information between agents and an idiosyncratic referential term that depends on individuals' past experiences. We structurally estimate the parameter governing the expectations-formation process, using the history of forecasts consumers would make in the model according to signals associated with their shopping experiences. As all respondents share current information, we control for the common forecast using time-fixed effects. The resulting estimation for the coefficient is less than zero, implying that consumers positively weigh their inflation history to the detriment of current unexpected news.[3]

We model the shared component following a standard signal-extraction procedure. Based on works exploring reference prices through consumers' shopping experiences (D'Acunto et al., 2021) and recent evidence showing that consumers use some food prices to form inflation expectations (D'Acunto & Weber, 2022), we use the lagged inflation rate of the food component of the CPI as a signal of the non-observed aggregate inflation variable.[4] Then, we construct a cohort-specific inflation expectations measure using (i) the already computed shared component, (ii) an idiosyncratic element related to the inflation history of the cohort, and (iii) the coefficient value estimated in the empirical section of the paper.

With our cohort-specific inflation expectations measure, we obtain model-based forecasts that closely match the inflation expectations observed in the data across cohorts and time. Regressing our model-based cohort-specific expectations on the individual expectations observed in survey data delivers a statistically significant coefficient of 0.888. Although the model-based inflation forecasts do not use micro-level information on inflation expectations, they predict consumers' survey data remarkably well. By using these two simple consumer references — shopping experience and history of inflation — we can effectively model survey households' expectations. This paper shows that consumer expectations surveys contain relevant and meaningful information that can be modeled.

As in Bianchi et al. (2021) and L'Huillier et al. (2021), we incorporate our proposed expectation formation process into a New Keynesian model.[5] Our proposed framework differs from and com-

[3]This result does not imply that agents cannot overreact to current news. Our empirical exercise shows that the proposed expectation modeling framework explains the heterogeneity observed across cohorts. However, the presence of a common component in the modeling allows for a common overreaction to some current news.

[4]Our findings are robust to using more specific signals such as CPI's food at home or dairy components, which, according to D'Acunto & Weber (2022), are the prices that US consumers primarily consider when forming inflation expectations.

[5]They include the diagnostic expectation model proposed by Bordalo et al. (2020) into a Dynamic Stochastic

plements these works. Bianchi et al. (2021) and L'Huillier et al. (2021) use a rational expectations operator, such that agents overreact to any news in a representativeness heuristic way (Kahneman & Tversky, 1972) and where full information characterizes the most likely scenario.[6] We model a history-based heuristic instead. We explicitly incorporate agents' history into the model and use it as their referential information. Due to this approach, we can exploit cohort-specific time-evolving history and show that consumers use this *availability* heuristic in their expectation formation process. Hence, agents' inflation experiences are over-sampled (Bordalo et al., 2023) as the past is over-extrapolated (Angeletos et al., 2021).

We then explore the macro implications of this micro-level heterogeneity in expectations, where we allow households to form expectations according to the proposed framework. In our model, while old generations have their expectations shaped mainly by their past, new generations are highly influenced by recent developments. We find that heterogeneous expectations anchor the aggregate inflation and output gap response to agents' history. At the same time, they also increase the duration of the effects of the shocks.[7] After an inflationary shock, the model produces hump-shaped expectations. This reaction is consistent with over-extrapolation as in Angeletos et al. (2021). Consumers react slowly to the inflationary shock in the first few periods, as they are tied to their reference from the steady state. This first reaction is consistent with randomized controlled trial evidence showing that consumers weigh their priors and do not react one-to-one to current or future inflation news (Weber et al., 2023). After witnessing higher inflation (and new cohorts entering the economy in a high-inflation environment), consumers get tied to this new inflationary reference and over-extrapolate the shock, overreacting in their forecast. From the aggregate results, we infer individual heterogeneous consumption and labor reactions.

We perform an optimal Taylor rule exercise where the central bank seeks to minimize the expected volatility of the economy by optimally choosing the parameters of a Taylor rule. When

General Equilibrium (DSGE) model. While Bordalo et al. (2020)'s model is an overreaction model, we use and extend their framework to allow for a negative distortion parameter, in their case, the diagnosticity parameter. In the following sections, we explain our microfundation. Notice that Bianchi et al. (2021) and L'Huillier et al. (2021) solutions also apply to our type of model.

[6] See Benjamin (2019) for a discussion on the implications of the assumptions of the diagnostic expectations model.

[7] While we could incorporate other forms of heterogeneity and biases related to experience-based mechanisms on the firm or the government side of the economy, in this paper, we introduce non-rational heterogeneous expectations only at the household level. We opt for this option mainly because of data availability motives.

we allow for heterogeneous expectations in the model, agents have long memories and remember current shocks far into the future. After a negative supply shock or a positive demand shock, the optimal response of the central bank is to be more active compared to the full information rational expectations case. In this manner, the monetary authority prevents inflation from rising and prevents agents from incorporating a high inflation episode into their memories.

This paper has important implications for explaining past inflation dynamics and learning about the future consequences of recent economic developments. Since 2021, a new cohort of consumers worldwide has been experiencing relatively high inflation for the first time. According to our findings, this high-inflation episode could have consequences in the medium run since consumers incorporate this episode into their history of inflation, adjusting future expectations. Our framework shows that accommodating high inflation produces higher and more persistent inflation expectations, which in turn generate a higher and more persistent inflation rate in the future. Our findings help us understand why inflation has persisted in the past, why consumers' inflation expectations are persistent today, what to expect from episodes of unusually high inflation, and how central banks should react to such episodes.

The rest of the paper is organized as follows. Section II discusses recent works on the topic. Section III provides empirical results regarding consumers' heterogeneity in inflation expectations. We empirically model inflation expectations depending on the history of inflation experienced by cohorts in Section IV. Section V discusses the aggregate implications arising from heterogeneous inflation expectations. Section VI shows results obtained for an optimal Taylor rule exercise. We analyze the high-inflation episode of 2021 through the lens of our theoretical model in Section VII. Finally, Section VIII concludes.

# II Literature review

Recent macroeconomic models show the relevance of heterogeneity in explaining aggregate fluctuations. However, the focus has been almost exclusively on households' financial constraints (Kaplan et al., 2018). Moreover, there are few studies on the aggregate role of expectations heterogeneity. Although surveys show significant heterogeneity across firms (Coibion et al., 2018) and households inflation expectations (Hajdini et al., 2022c), few works study its macroeconomic implications. A notable exception is Afrouzi (2020), which shows that heterogeneity in firm-level inflation expectations, coming from different levels of attention due to endogenous information acquisition on competitors' beliefs, amplifies monetary non-neutrality. Our paper focuses on the heterogeneity of expectations from the household side. In our framework, households' heterogeneous inflation expectations anchor the response of aggregate variables to agents' memories, increasing the persistence of the effects of shocks. Therefore, an energetic reaction from monetary authorities prevents current high inflation and prevents agents from incorporating high-inflation episodes into their memories, thus preventing higher future inflation expectations.

Recent empirical literature shows that households' inflation expectations depart from full information rational expectations and are heterogeneous. Relevant to our paper, Malmendier & Nagel (2016) document that households present learning from past inflation mechanisms when forming inflation expectations. People who have experienced higher inflation rates in the past have higher future inflation expectations. Therefore, heterogeneity of inflation expectations naturally arises due to different experiences with past inflation rates. Malmendier et al. (2021) discuss similar results, while other works focus on exploring other heterogeneity sources across firms and consumers (Hajdini et al., 2022c).

Households' inflation expectations depend on more than just past experiences. Evidence shows they also respond to other variables such as professional forecasts (Carroll, 2003), prices exposure (D'Acunto et al., 2021), and socioeconomic characteristics (D'Acunto et al., 2022), among others.[8] This departure from full information rational expectations and the presence of heterogeneity is relevant since households' inflation expectations affect a broad set of households' decisions (Roth & Wohlfart, 2020; Hajdini et al., 2022b). For instance, Malmendier & Nagel (2016) shows that inflation expectations influence individuals' financial decisions, while Coibion et al. (2023) confirms that inflation expectations partly determine households' spending on durable goods.

The model proposed in this paper features monetary policy, a learning from the past mechanism, and overlapping generations, connecting the paper to several strands of the theoretical literature. First, our model closely follows the literature enclosing behavioral New Keynesian models such as the ones proposed by Branch & McGough (2010), Gabaix (2020), Gáti (2020), and Pfäuti &

[8]Although our focus is on the household side of the economy, evidence shows that professional forecasters depart from rational expectations too (Coibion & Gorodnichenko, 2015; Bordalo et al., 2020; Gáti, 2020). On the other hand, due to data availability, evidence on firms' expectations is notably scarce (Candia et al., 2022).

Seyrich (2023); as well as behavioral overlapping generations models as in Adam (2003). By considering overlapping generations in a New Keynesian context, we relate, for instance, to Gali (2021). By stating that cohorts show heterogeneity in expectations because of different experiences, we connect to papers unrelated to monetary policy but where different cohorts have different beliefs about the future, such as Collin-Dufresne et al. (2017) and Kuchler & Zafar (2019).

Our model of non-rational and heterogeneous expectations is also inspired by the diagnostic expectations literature, as introduced in Bordalo et al. (2018, 2019, 2020).[9] Bianchi et al. (2021) and L'Huillier et al. (2021) provide recent applications of this framework in macro monetary settings. We use that setting to incorporate our findings into a general equilibrium model in a tractable way.

Besides diagnostic expectations, a large body of literature analyzes the implications of departing from the full information rational expectations assumption. Among the main examples, we include the set of papers related to the imperfect information approach (Mankiw & Reis, 2011), the complex systems/animal spirits/heuristic approach (De Grauwe, 2011; Jump et al., 2019), the sticky information approach (Mankiw & Reis, 2002), and the adaptive learning approach (Marcet & Sargent, 1989; Evans & Honkapohja, 2001). However, only some of these papers have studied how heterogeneity in expectations arises and its macroeconomic consequences.

Our findings are closely related to those of Malmendier & Nagel (2016), who use adaptive learning to approximate cohorts' heterogeneous inflation expectations. Instead, we opt for a constant forecasting revision Kalman filter model. While we also rely on constant gain, the selection of parameters in our framework is primarily data-driven. Our approach allows us to incorporate a current shared forecast component using a standard Kalman filter and a structure incorporating past inflation experiences.

Our paper shows that cohorts do not adjust their expectations differently in response to current inflation news. Our results suggest that younger cohorts implicitly put more weight on current information but adjust to the news data similarly to older cohorts. In other words, younger cohorts do not react more strongly to inflation news than older cohorts. In that sense, our modeling follows Malmendier & Nagel (2016) by incorporating a method consistent with the availability heuristic by using past experiences and allowing agents to use the new information to form expectations. Fol-

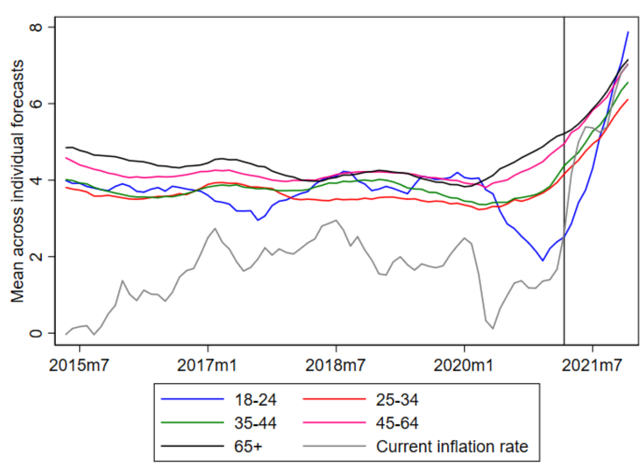
[9] Our approach is also related to broader signal extraction/noisy information approaches such as the ones proposed in Woodford (2001) and Blanchard et al. (2013), among others.

lowing recent evidence showing that agents form expectations based on their shopping experiences (D'Acunto et al., 2021; D'Acunto & Weber, 2022), we use a Kalman filter approach where agents get signals from current food prices to model the expectation formation process. In addition, our approach is flexible enough to be incorporated into a general equilibrium framework as some other recent studies (Bianchi et al., 2021; L'Huillier et al., 2021).

# III Empirical facts

This section reviews some empirical facts related to heterogeneous inflation expectations at the household level. We show how these expectations are correlated with past experiences regarding the aggregate inflation variable. These empirical facts motivate and guide the theoretical model of the paper.

Figure 1: Average 12-months-ahead inflation expectations.




**Note**: The figure shows the 12-month moving average for the 10 percent and 90 percent trimmed mean for each cohort using the point forecast. We use population weights. Data goes from June 2013 to December 2021. Ages correspond to the interviewee's age at the time of the survey. The vertical line denotes March 2021. The current inflation rate measure is based on the monthly YoY percentage variation of the CPI.
**Source**: Survey of Consumer Expectations, Federal Reserve Bank of New York.

Malmendier (2021) and D'Acunto et al. (2022) document that consumers' experiences influence their inflation expectations. Thus, individual experiences are a source of expectation heterogeneity. This paper focuses on how aggregate inflation experiences influence idiosyncratic inflation expectations, as in Malmendier & Nagel (2016). In the US economy, this heterogeneity in inflation expectations became more evident after the high-inflation episode of 2021, when inflation surged after 30 years of low and stable rates.

For this section, we use data from the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York. This data set is a US-wide rotating panel with information between March 2013 and December 2021, where each respondent is surveyed for a maximum of 12 contiguous months. This data set is handy for our purposes because it provides high-frequency data on American households' inflation expectations in two different periods of the US economy. In particular, we focus our analysis on respondents' 12-months-ahead point forecast. The 12-months-ahead inflation rate is computed as the inflation rate existing between the current month and 12 months after the current month.[10]

**Fact 1: Inflation expectations are heterogeneous across cohorts.**

Figure 1 shows the mean 12-months-ahead inflation forecast by cohort. The heterogeneity across cohorts is evident. The oldest (65+) and the second oldest cohort (45-64) have higher mean inflation expectations throughout most of the sample. Those cohorts experienced a period of high inflation in the 60s, 70s, and early 80s. Regarding inflation forecast value, these cohorts are followed by intermediate cohorts (25-34 and 35-44), who experienced the stable and low inflation rates of the 90s, 00s, and 10s. Finally, the youngest cohort (18-24) shows the most volatile mean, following the current inflation rate most of the time. The mean value of this cohort notably increased after the high-inflation episode of 2021, surpassing older cohorts' expectations.

**Fact 2: Inflation experiences are clustered by age.**

Figure 2 plots the average lifetime inflation rate people have experienced according to their age in the years 2020 and 2021. In the US, the average lifetime inflation rates are clustered by age.

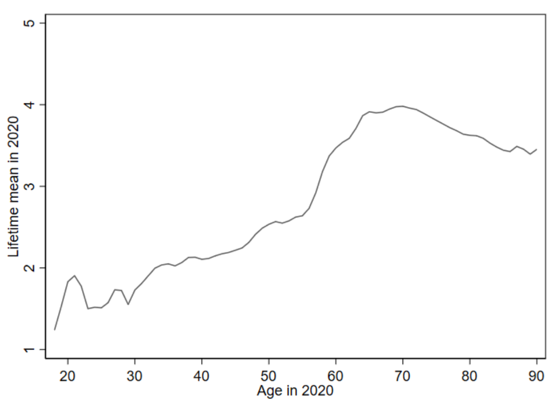
The heterogeneity of average experienced inflation rates across cohorts results from the different inflation-related events Americans have gone through. Older cohorts have experienced events such as the Great Inflation period (1965-1982), characterized by high and persistent inflation. Thus,

[10] Specifically, consumers are asked to answer first the following question: "The next few questions are about inflation. Over the next 12 months, do you think that there will be inflation or deflation?" After indicating if they forecast inflation or deflation, they are asked to give a numerical answer to the following question:"What do you expect the rate of inflation/deflation to be over the next 12 months? Please give your best guess. "

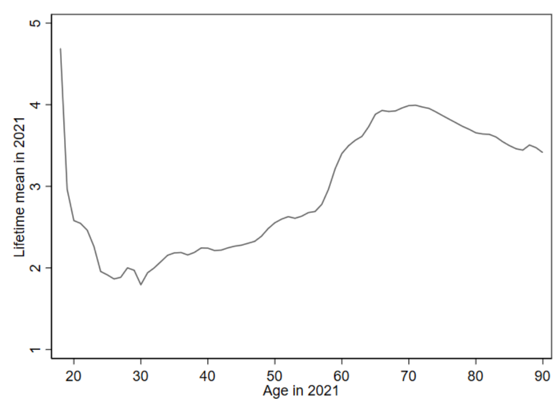
these cohorts have a higher lifetime average inflation rate, regardless of the year we calculate. Meanwhile, intermediate cohorts have experienced low and stable inflation rates throughout the 80s, 90s, 00s, and 10s. Therefore, they present lower values on the lifetime average inflation rate. For older and intermediate cohorts, experiencing the high-inflation episode of 2021 did not significantly affect their lifetime average inflation rate.

In contrast, youngest cohorts show a significant change between 2020 and 2021. Until 2020, the youngest cohorts had not experienced high inflation, showing low lifetime average inflation rates. However, after being exposed to the high-inflation episode of 2021, their lifetime average inflation rate dramatically increased.

Figure 2: Lifetime average inflation rate among respondents.




(a) 2020




(b) 2021

**Note**: The figure shows the mean of the monthly YoY CPI-based inflation rate that people of the age indicated in the years 2020 and 2021 have experienced in their lifetimes, starting when they were age 18.
**Source**: Bureau of Labor Statistics.

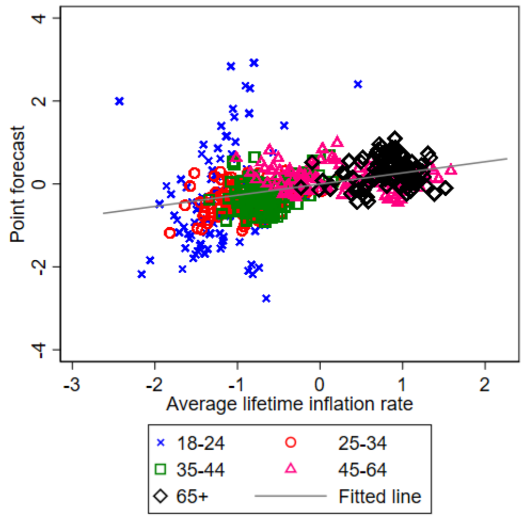
**Fact 3: A higher average lifetime inflation rate is correlated with a higher point forecast.**

Tying together both previous empirical facts, Figure 3 shows that people who have experienced higher average inflation rates during their lifetimes, when surveyed, tend to give a higher inflation point forecast.[11] We formally test this result in Table 1. Columns 3 and 4 of this table conclude that the inflation experienced significantly affects individuals' inflation expectations, even after controlling for the current environment and individual characteristics.

[11]We control for observable characteristics of the respondent except for the age and period variables.

This fact provides empirical support for the literature on learning from past experiences (Malmendier & Nagel, 2016; Malmendier, 2021; Malmendier et al., 2021; Malmendier & Wachter, 2022) pointing to a possible source of heterogeneity in inflation expectations: past experiences with the aggregate inflation variable.

Figure 3: Inflation point forecast and average lifetime inflation.




**Note**: The figure shows binned scatterplot across lifetime average inflation bins. We residualized the variables by respondent gender and commuting zone. Data goes from June 2013 to December 2021. Ages correspond to the interviewee's age at the time of the survey. The average lifetime inflation rate measure is based on the monthly YoY percentage variation of the CPI.
**Source**: Survey of Consumer Expectations, Federal Reserve Bank of New York and Bureau of Labor Statistics.

While the evidence we provide here is for the US economy, in Appendix G, we find similar evidence for a panel of European countries. With this European data set, we show that the pattern (i) is present beyond the US and (ii) does not arise from cohorts' systematic characteristics but because of cross-country heterogeneous inflation experiences. In the panel of countries we use, we observe different inflation histories across countries that are not necessarily similar to the US experience. Again, we find that average inflation experience positively relates to individual inflation expectations, even after including country-time fixed effects and, more importantly, cohort fixed effects. This last set of fixed effects controls for the fact that cohorts can have biases because of motives related to their age. In this sense, Hajdini et al. (2022a) find similar evidence using a survey for a panel of countries.

Table 1: Effects of current and experienced inflation on inflation expectations

| Dep. var.: Inflation expectations | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Average lifetime inflation | 0.325*** | 0.259*** | 0.293*** | 0.244*** |
| | (0.026) | (0.065) | (0.024) | (0.023) |
| Current inflation | 0.523*** | 0.669*** | | |
| | (0.017) | (0.119) | | |
| Cohort 25-34 | | 0.008 | | |
| | | (0.342) | | |
| Cohort 35-44 | | -0.048 | | |
| | | (0.328) | | |
| Cohort 45-64 | | 0.082 | | |
| | | (0.348) | | |
| Cohort 65+ | | 0.175 | | |
| | | (0.352) | | |
| Current inflation × 25-34 | | -0.202 | | |
| | | (0.131) | | |
| Current inflation × 35-44 | | -0.130 | | |
| | | (0.128) | | |
| Current inflation × 45-64 | | -0.119 | | |
| | | (0.124) | | |
| Current inflation × 65+ | | -0.187 | | |
| | | (0.127) | | |
| Time FE | No | No | Yes | Yes |
| Controls | No | No | No | Yes |
| Observations | 105,413 | 105,413 | 105,413 | 105,399 |
| R-squared | 0.057 | 0.057 | 0.091 | 0.196 |

**Note**: Table shows regressions where the dependent variable is inflation expectations according to the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York. Column 1 shows controls by the average lifetime inflation of respondents of a given age at each period of time and the last inflation measure. Column 2 follows (1) but adds cohort fixed effects and the interaction of those cohort fixed effects with the current inflation. Column 3 follows Column 1 but adds time fixed effects and, hence, omits the current inflation variable. Column 4 follows Column 1 but adds time fixed effects and demographic controls. The demographic controls are income, gender, Hispanic origin, race, educational level, numerical proficiency, and commuting zone. The current inflation rate measure is based on the monthly YoY percentage variation of the CPI. Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1. Standard errors clustered by age. The dependent variable is trimmed, dropping the lower and upper 10 percent of answers in each period.

**Fact 4: After controlling for the average lifetime inflation rate, younger cohorts do not react more strongly to inflation news than older cohorts.**

We test whether younger generations react more strongly to the current economic environment after controlling for their average lifetime inflation. This exercise aims to inform the modeling approach we will take. The marginal contribution of new information for younger cohorts is higher, as they have a shorter history. Assuming constant weights implies that new information will matter more, especially in the context of persistent inflation. From a modeling perspective, we want to know whether cohorts react differently to news, conditional on their cohort-dependent marginal contribution of new information. These findings help us to discriminate between models.

We test this hypothesis through individual-level regressions presented in Table 1. Similar to the results of other papers using information treatment (Hajdini et al., 2022b), Column 1 of this table shows that all individuals react to current inflation events. These results also confirm the existence of a positive relationship between the inflation forecasts and average lifetime inflation rates, as we saw previously in Figure 3, even after considering current inflation.

To study whether there are different reactions across cohorts, we run regressions that consider interactions of current inflation with a cohort indicator variable. Column 2 of Table 1 shows the corresponding results. After controlling for average lifetime inflation, the interaction term has no statistically significant effect. There are no different reactions to current inflation news across cohorts. We confirm the finding by performing an F-test where the null hypothesis is that all interactions are jointly equal to zero. The test gives a p-value of 0.39, so we cannot reject the null hypothesis. In addition, this conclusion holds in a sample of European countries, as we show in Table A7 of Appendix G. Together, these results suggest that the main source of heterogeneity across cohorts comes from different past experiences regarding inflation.[12]

# IV A simple model with heterogeneous expectations

In this section, we propose a history-based expectations-augmented Kalman filter as the process by which agents form their inflation expectations. We begin with a simple model that provides a good starting point where differences in agents' personal experiences do not imply heterogeneity in expectations. Given the absence of private information, we show that the observed heterogeneity cannot arise from a standard Kalman filter. Then, by introducing a history-based expectations-augmented Kalman filter, we explain how the private inflation history distorts the expectations, producing inflation forecast heterogeneity. Moreover, we estimate the corresponding distortion parameter and close the section by comparing the heterogeneous rates of inflation expectations generated by our proposed framework and those observed in the data and presented in Figure 1.

[12]Furthermore, in Appendix D we assume the structure of a standard Kalman filter and provide empirical evidence that provides support for the hypothesis of different cohorts not reacting differently to inflation signals.

## IV.1 Standard Kalman filter

### IV.1.1 Setup

The economy is populated by different cohorts indexed by $i$. These cohorts are heterogeneous in their dates of birth and the inflation history they have experienced. Since there is no heterogeneity within cohorts, a single representative agent summarizes the situation of each one of these groups. In a given period $t+1$, the level of inflation $\pi_{t+1}$ is defined according to the following random walk process[13]

$$\pi_{t+1} = \pi_t + \varepsilon_t,$$

where $\varepsilon_t$ is a normally independent and identically distributed inflation shock. We assume that, in a given period $t$, agents wish to forecast the future inflation rate $\pi_{t+1}$, but they only observe a noisy signal of this variable. In other words, the agents face a standard signal extraction problem. To simplify the analysis, in a given period $t$, we assume that the signal $s_t$ is defined as

$$s_t = \zeta \pi_{t+1} + \upsilon_t,$$

where the coefficient $\zeta \geq 0$ denotes the pass-through existing between the unobserved variable $\pi_{t+1}$ and its corresponding signal $s_t$, and $\upsilon_t$ is a signal noise. We assume that this noise is a normally independent and identically distributed variable. Moreover, we allow for a non-zero covariance between both shocks to consider the existence of some elements causing correlated movements in both the observed signal and the unobserved variable. Therefore, we have

$$\begin{pmatrix} \varepsilon_t \\ \upsilon_t \end{pmatrix} \sim \mathcal{N}\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\upsilon} \\ \sigma_{\varepsilon\upsilon} & \sigma_\upsilon^2 \end{pmatrix} \right).$$

As a further simplification, we assume the model has no private information. In other words, all of the agents receive precisely the same signal. Since the agents face a standard signal extraction

[13] We opt for a random walk process instead of a first-order auto-regressive (AR(1)) specification because the data cannot reject the hypothesis that the monthly inflation rate has a unit root. We provide a more thorough discussion in Appendix C. Also, see Pivetta & Reis (2007) for a discussion on the very high persistence of the (quarterly) inflation rate in the US. To complete our analysis, in Appendix E, we show the model's results when the inflation series follows an AR(1) process. These results are very similar to those found under the random walk assumption.

problem, we assume they generate inflation forecasts using this variable's conditional expected value. More precisely, given their information set in period $t$, the agents apply a linear Kalman filter to forecast inflation in period $t+1$. Therefore, the inflation's predicted value is given by

$$\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right] = (1-\zeta K)\,\mathbb{E}_{i,t-1}^{KF}\left[\pi_{t+1}\right] + Ks_t, \tag{1}$$

where $K$ denotes a standard Kalman gain.[14] Conditional on agents' past and current signals, the Kalman filter approach allows us to characterize the forecasted distribution of the unobserved variable $\pi_{t+1}$ in any period $t$. When the signal is perfectly revealing about the true state of the variable, we observe $\zeta = 1$, $\upsilon_t = 0$, and $\sigma_{\varepsilon\upsilon} = 0$. Therefore, we conclude $K = \zeta K = 1$ and $\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right] = s_t = \pi_{t+1}$. The presence of a signal noise induces $K \in [0,1)$ even without a correlation between both error terms.

Regarding long-run values of inflation expectations, from the Kalman-based forecast equation and using the random walk structure associated with the inflation variable, we conclude that given $h \geq 1$, we must have

$$\mathbb{E}_{i,t}^{KF}\left[\pi_{t+h}\right] = \mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right].$$

Finally, and considering the parameter $\gamma = (1-\zeta K) \in [0,1]$, the Kalman filter prediction can be written recursively as

$$\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right] = \gamma^{t+1}\mathbb{E}_{i,-1}^{KF}\left[\pi_0\right] + K\sum_{j=0}^{t}\gamma^{t-j}\left(\zeta\pi_{j+1} + \upsilon_j\right)$$

Therefore, using this simple version of the model, we conclude that higher values of past inflation imply a higher forecasting value of this same variable. However, agents' personal experiences are not associated with heterogeneity in expectations. According to this model, agents who lived

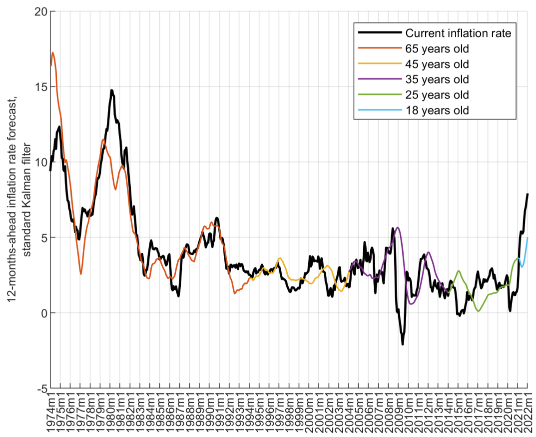
[14] As usual in the literature, this signal-to-noise ratio is defined such that it minimizes the variance of the prediction error associated with the unobserved variable, i.e., $\pi_{t+1} - \mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right]$. The Kalman gain that solves this optimization problem depends on the covariance existing between the error associated with the observed signal and the unobserved variable and the constant $\Sigma_{t+1|t-1} = \mathbb{V}\text{ar}\left[\pi_{t+1} - \mathbb{E}_{i,t-1}^{KF}\left[\pi_{t+1}\right]\right]$. See Cheung (1993) for an example of a Kalman gain that considers these terms. Regarding the constant $\Sigma_{t+1|t-1}$, it can be shown that it must satisfy $\left(\Sigma_{t+1|t-1} - \sigma_\varepsilon^2\right)\left(\zeta^2\Sigma_{t+1|t-1} + \sigma_\upsilon^2 + 2\zeta\sigma_{\varepsilon\upsilon}\right) - \left(\sigma_\upsilon^2\Sigma_{t+1|t-1} - \sigma_{\varepsilon\upsilon}^2\right) = 0$. We opt for a constant Kalman gain as it is a convenient assumption for the dynamic model explored in the following sections. However, we find similar results when we use time-varying gains.

through episodes of high inflation forecast an inflation value identical to those who lived through episodes of low inflation. Given a starting point assumption where every agent observes the initial level of inflation, i.e. $\mathbb{E}_{i,-1}^{KF}[\pi_0] = \pi_0$ for every agent $i$, we conclude that $\mathbb{E}_{i,t}^{KF}[\pi_{t+1}] = \mathbb{E}_{t}^{KF}[\pi_{t+1}]$ must hold for every agent. In what follows, to simplify the analysis, we assume $\zeta = 1$.

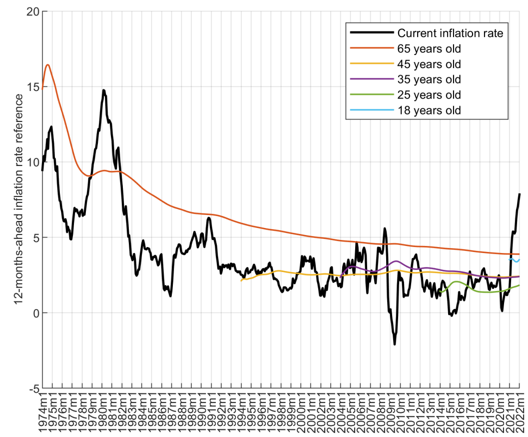
To identify an appropriate signal for the empirical counterpart of the expectations formation model, we follow the evidence presented in D'Acunto et al. (2021), which shows that agents use their consumption experience to form expectations. More specifically, Campos et al. (2022), using the University of Michigan's Survey of Consumers, concludes that consumers highly weigh CPI's food components when forming inflation expectations. Dietrich (2022) finds similar evidence using different data sources. Although differences in the composition of individual expenditures could lead to differences in perceived inflation (D'Acunto et al., 2021), according to data from the US Bureau of Labor Statistics, the share of food expenditure has no relevant variation across US cohorts. This fact supports our assumption of a common signal on the underlying aggregate inflation represented by the food component of the CPI. Finally, D'Acunto & Weber (2022) show that consumers use food prices as a reference to form inflation expectations. Therefore, we use the food component of the CPI as a common inflation signal for all consumers.

Because of timing issues, we use the lagged year-over-year percentage variation of the CPI's food component. Consumers use past changes in food prices as a signal to forecast future aggregate inflation. For example, suppose that in December, an agent forecasts aggregate inflation for the next 12 months, and we presume that consumers make this prediction at the beginning of the month. In that case, we assume this agent considers November's food inflation to forecast. Therefore, for the empirical counterpart of the expectations formation model, we assume $s_t = \pi_{t-1}^{food}$ where $\pi_{t-1}^{food}$ denotes food inflation in period $t-1$. D'Acunto & Weber (2022) show that this behavior is consistent with consumers' use of food prices to form expectations, as they show that consumers weigh changes of observed reference food prices, in their case milk, to form inflation expectations. Hence, we use annual changes in the CPI's food component for the baseline specification primarily because of data availability motives and consistency with recent evidence. In Appendix F, we show that our results are robust to alternative signals also consistent with the recent empirical evidence on reference prices.

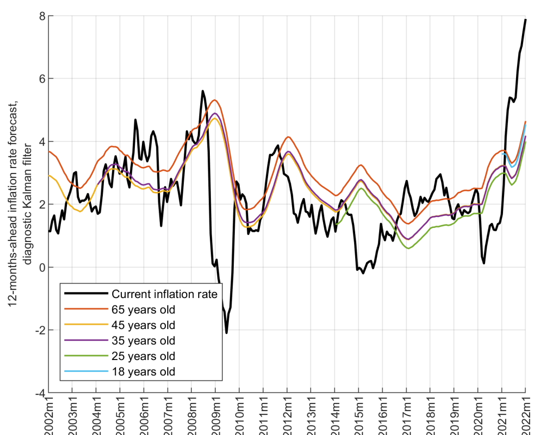
Figure 4: History-based Kalman-filter-based inflation forecasts by cohort




(a) Standard Kalman-filter-based inflation forecasts




(b) Inflation rate reference




(c) History-based Kalman-filter-based inflation forecasts

**Note**: Panel (a) shows the Kalman filter forecast for the common component for selected cohorts, differentiated by their age in 2021. Panel (b) shows the references for selected cohorts obtained according to the Kalman filter and given the history of inflation experienced by the corresponding age group. Panel (c) shows forecasts for selected cohorts according to the Kalman-filter-augmented expectations and considering the estimate for $\theta$ from Column 1 of Table 2. Selected cohorts are differentiated by their age in 2021. We further assume that each cohort starts forecasting when they become 18 years old. The current inflation rate measure is based on the monthly YoY percentage variation of the CPI.

Using monthly data on aggregate and food inflation, we obtain $\sigma_\varepsilon^2 = 0.15$, $\sigma_\upsilon^2 = 4.09$ and $\sigma_{\varepsilon\upsilon} = -0.03$, concluding $K = 0.1751$.[15]

We now perform a forecasting exercise using monthly inflation data and distinguishing agents by cohorts. Given the recursive structure of the Kalman filter and to initialize the cohort-specific forecasting process, we assume that in the period in which the cohort representative agent reaches

[15] We detail how we obtain this calibration in Appendix C, while Appendix D provides empirical support for the Kalman gain being constant across cohorts.

adulthood and begins forecasting, she uses the previous period's Kalman filter expected value as a starting point. We denote the period when agent $i$ starts forecasting as period $k_i$. Given the starting point assumption where the initial level of inflation is common knowledge, we have $\mathbb{E}^{KF}_{i,k_i-1}[\pi_{k_i}] = \mathbb{E}^{KF}_{k_i-1}[\pi_{k_i}]$ for every agent $i$. Panel (a) of Figure 4 presents the 12-month-ahead inflation forecasts of different cohorts produced according to the standard Kalman filter. This figure plots the actual inflation rate and the forecast made by some selected cohorts. The standard Kalman filter cannot generate the heterogeneous pattern in expectations observed in the data (i.e., Figure 1). Under this framework, inflation expectations evolve following an identical process across cohorts. Therefore, we need to move to a more sophisticated framework to replicate the facts observed in the data.

## IV.2 History-based Kalman filter

### IV.2.1 Setup

In this section, we depart from the standard Kalman filter framework. We adopt a model of non-Bayesian beliefs where consumers revise their forecast using a reference based on their experienced inflation history.[16] In this framework, consumers receive signals and compare them to their history of forecasts for the next period. By making that comparison when forecasting, consumers can put more weight on the new information they receive or on their past history-specific information. Our framework allows for both scenarios.

When consumers put more weight on the likelihood of new information for forecasting, our setting would be as in Bordalo et al. (2019), related with the representativeness heuristic. When consumers put more weight on their past history-specific information for forecasting, it would be associated with the availability heuristic (Tversky & Kahneman, 1973). That is, when forming expectations, consumers use their available information related in this case to their history, instead of sampling the complex current information that is needed to form future expectations about the economy. This version of the model can also be related to the conservatism bias (Edwards, 1968),

[16]We can think of our framework as a generalization of the diagnostic expectations framework. The generalization comes due to the fact that we do not restrict the parameter $\theta$ to be strictly positive. Thus, in terminology of the diagnostic expectations, we may find overreaction ($\theta$>0), underreaction ($\theta$<0), or neither ($\theta$=0). A similar generalization of diagnostic expectations theory is discussed in Liao (2023).

as consumers put less weight on the signal when forming their beliefs.

This setting aims to model the source of the heterogeneity in inflation expectations, not to make a general statement about expectations formation processes. This source of heterogeneity is relevant, consistent across surveys of expectations, and it complements other findings on inflation expectation formation. The flexibility of this framework allows us to connected to our empirical setting and to learn about how people use their history of inflation when they form expectations.

We denote the true conditional distribution of the unknown inflation variable in a given period $t$ as $f(\pi_{t+1} \mid \mathscr{I}_t)$, where the term $\mathscr{I}_t$ denotes the common set of information available in that period. Given this definition, we assume that the inflation's belief distribution for agent $i$ is given by

$$f_{i,t}^{\theta}(\pi_{t+1}) = f(\pi_{t+1} \mid \mathscr{I}_t) D_{i,t}^{\theta}(\pi_{t+1}) Z_{i,t}, \tag{2}$$

where

$$D_{i,t}^{\theta}(\pi_{t+1}) = \left[ \frac{f(\pi_{t+1} \mid \mathscr{I}_t)}{f\left(\pi_{t+1} \mid \mathscr{I}_{i,t}^{ref}\right)^{\frac{1}{(t-k_i)}}} \right]^{\theta}.$$

In this setup, the parameter $\theta \in \mathbb{R}$ governs the level of distortion that the likelihood ratio $\left[D_{i,t}^{\theta}(\pi_{t+1})\right]^{\frac{1}{\theta}}$ introduces into agents' beliefs. The normalizing parameter $Z_{i,t}^{-1} = \int f_{i,t}^{\theta}(\pi_{t+1})\, d\pi_{t+1}$ ensures that the distribution $f_{i,t}^{\theta}(\pi_{t+1})$ integrates to one in every period $t$ and for every agent $i$. Each agent compares the current common information set $\mathscr{I}_t$ against her referential information set $\mathscr{I}_{i,t}^{ref}$. Later, we assume that this idiosyncratic referential information set relates to each agent's past inflation experiences. As mentioned above, the parameter $\theta$ captures the level of distortion associated with the model. In the standard Kalman filter framework, we have $\theta = 0$, which implies $D_{i,t}^{\theta}(\pi_{t+1}) = 1$. When this is the case, there is no distortion in beliefs, and $f_{i,t}^{\theta}(\pi_{t+1}) = f(\pi_{t+1} \mid \mathscr{I}_t)$. When $\theta \neq 0$, beliefs are distorted, as we discuss below.

Notice that the no private information assumption implies that the information associated with the true conditional distribution is equal for everyone. Therefore, under the proposed signal extraction framework, in any period $t$, the expected value associated with the true conditional distribution of the unknown inflation variable is common to every agent and corresponds to $\mathbb{E}_t^{KF}[\pi_{t+1}]$. Given the normality assumption on the error term of the inflation process, the true conditional distribution satisfies $f\left(\pi_{t+1} \mid \mathscr{I}_t\right) \sim \mathscr{N}\left(\mathbb{E}_t^{KF}\left[\pi_{t+1}\right], \sigma_\pi^2\right)$. As we explain below, this normality result implies that the distribution associated with the referential information set $\mathscr{I}_{i,t}^{ref}$ is normal too. Given both of these results, we can show that agent $i$'s distribution satisfies $f_{i,t}^{\theta}\left(\pi_{t+1}\right) \dot{\sim} \mathscr{N}\left(\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+1}\right], \sigma_\pi^2\right)$, where the mean value has the following linear structure

$$\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+1}\right] = \mathbb{E}_t^{KF}\left[\pi_{t+1}\right] + \theta\left(\mathbb{E}_t^{KF}\left[\pi_{t+1}\right] - \mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right]\right), \tag{3}$$

where $\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+1}\right]$ is the distorted forecast associated with the belief distribution $f_{i,t}^{\theta}\left(\pi_{t+1}\right)$, and $\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right]$ is the expected value obtained according to the distribution associated with the referential information set $\mathscr{I}_{i,t}^{ref}$.[17] We define this linear composition of the standard Kalman filter as our history-based-augmented Kalman filter.

Up to this point, our variables are history-independent. Remember that the standard Kalman filter is Markovian since it only requires the previous period's beliefs to be computed. However, this simple filter does not reproduce our set of empirical facts. We now introduce a role for past experiences through the reference term $\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right]$. For the cohort $i$'s representative agent, we define the reference term as

$$\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right] = \frac{\sum_{j=1}^{t-k_i} \mathbb{E}_{i,t-j}^{KF}\left[\pi_{t+1}\right]}{\left(t-k_i\right)}, \tag{4}$$

where $k_i$ denotes the period in which cohort $i$ starts forecasting. Notice that the reference term contains all past expectations of this cohort-specific agent about future inflation. Since the reference term uses food prices as a signal, it contains the history of how consumers of a given cohort used the food prices they were exposed to forecast aggregate inflation. In Panel (b) of Figure 4, we show how the inflation rate reference defined according to Equation 4 evolves for some selected cohorts. Older cohorts, which have gone through episodes of higher inflation during their lifetimes, have higher reference points when compared to younger cohorts that have not experienced relevant inflationary episodes.

Now, we turn the discussion into the parameter $\theta$. When $\theta > 0$, agents "overreact" to the current information they receive relative to their prior beliefs. For instance, if agents receive news about inflation being different from what they expected in the past, they will overreact to this news by

[17] See Appendix B for a more detailed derivation.

adding more weight to current information, under-weighting their inflation history. If $\theta < 0$, agents "underreact" to the information just received, placing more weight on their references relative to current information. In this case, if agents observe inflation values different than their priors, they will tend to anchor their current expectations to their past beliefs. As discussed in Section III, recent economic developments show that older consumers had higher inflation expectations when aggregate inflation was low, such as the period between 2010 and 2020. Meanwhile, when inflation was high in 2021, consumers reacted moderately, not increasing their expectations significantly. This idea goes in line with agents anchoring their expectations to their experience or, in our framework, with a parameter satisfying $\theta < 0$. Lastly, when $\theta = 0$, there are no distortions in beliefs and $\mathbb{E}^{\theta}_{i,t}[\pi_{t+1}] = \mathbb{E}^{KF}_{t}[\pi_{t+1}]$ for every cohort $i$.

Given the random walk structure assumed for the inflation variable, and considering $h \geq 1$, we conclude $\mathbb{E}^{KF}_{t}[\pi_{t+h}] = \mathbb{E}^{KF}_{t}[\pi_{t+1}]$. Since the reference term is a linear composition of expected values associated with the true conditional distribution, we must have $\mathbb{E}^{ref}_{i,t}[\pi_{t+h}] = \mathbb{E}^{ref}_{i,t}[\pi_{t+1}]$. Therefore, when $h \geq 1$, we observe $\mathbb{E}^{\theta}_{i,t}[\pi_{t+h}] = \mathbb{E}^{\theta}_{i,t}[\pi_{t+1}]$.

#### IV.2.2 Estimation and forecasting exercise

In this section, we estimate the parameter $\theta$ directly from the data. Under our current assumptions, from Equation 3 we know that the cohort $i$'s forecast for period $t+12$ corresponds to

$$\mathbb{E}^{\theta}_{i,t}[\pi_{t+12}] = (1+\theta)\,\mathbb{E}^{KF}_{t}[\pi_{t+12}] - \theta\mathbb{E}^{ref}_{i,t}[\pi_{t+12}]. \tag{5}$$

Since the first term is common across all cohorts, it can be captured by a time-fixed effect $\gamma_t$. Regarding the distorted expectation, we use data on the 12-month-ahead forecast of each agent $m$ from cohort $i$ presented in the SCE survey. We denote this variable as $\mathbb{E}^{SCE}_{m,i,t}[\pi_{t+12}]$. Finally, to compute $\mathbb{E}^{ref}_{i,t}[\pi_{t+12}] = (t-k_i)^{-1}\sum_{j=1}^{k_i}\mathbb{E}^{KF}_{i,t-j}[\pi_{t+12}]$ we return to the standard Kalman filter from Section IV.1 and recover the different terms $\mathbb{E}^{KF}_{i,t-j}[\pi_{t+12}]$, which are optimal forecasts under the given setup. Given this setting, we run the following regression

$$\mathbb{E}^{SCE}_{m,i,t}[\pi_{t+12}] = \gamma_t + \varphi\mathbb{E}^{ref}_{i,t}[\pi_{t+12}] + \varepsilon_{m,i,t}. \tag{6}$$

We present the results in Table 2. Column 1 shows our main specification, from which we obtain $\widehat{\theta} = -\widehat{\varphi} = -0.317$. Because the estimated parameter satisfies $\widehat{\theta} < 0$, we conclude that when agents forecast, they positively weigh their reference sets (i.e., their past inflation history, their priors coming from their experiences with food prices). In the remaining columns of Table 2, we show that the negative coefficient is robust to the inclusion of control variables.[18]

This result contrasts with the diagnostic expectations literature (for instance, Bordalo et al. 2020), where a positive $\theta$ is the usual result using a similar empirical structure. However, this positive $\theta$ result is based on a history-based expectations formation process with a different reference point. Instead of using agents' past inflation history, the diagnostic expectations literature usually uses the one-period lagged information set as their reference point. Moreover, their finding is also conceptually different from ours, as most of the previous empirical literature on diagnostic expectations relies on surveys of professional forecasters, who are better informed about the economy than the households surveyed in the SCE.[19]

Table 2: Parameter estimation

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $\mathbb{E}_{i,t}^{ref}[\pi_{t+12}]$ | 0.317*** (0.030) | 0.354*** (0.028) | 0.260*** (0.030) | 0.224*** (0.026) |
| Time FE | Yes | Yes | Yes | Yes |
| Controls | No | Gender, commuting zone | Gender, commuting zone, HH income | Gender, commuting zone, HH income, educational degree |
| Observations | 101,262 | 101,245 | 101,245 | 101,245 |
| R-squared | 0.092 | 0.148 | 0.169 | 0.183 |

**Note**: Table shows results of Regression 6. $\mathbb{E}_{i,t}^{ref}[\pi_{t+12}]$ is the reference constructed for a respondent of age *i* as explained in the main text. Column 1 has only a time fixed effect as an additional control. Columns (2), (3) and (4) add different levels of controls. Robust standard errors in parentheses. Standard errors clustered by age. Dependent variable trimmed at 10 percent and 90 percent in each period. *** p<0.01, ** p<0.05, * p<0.1.

The value of the parameter $\theta$ is also conditional on the expectation formation process. In Bor-

[18] In Figure A2 of Appendix A we show the results of estimating Equation 6 but leaving one cohort out of the sample in successive regressions. We find that the coefficient of interest remains stable throughout the successive estimations and, thus, does not depend on the inclusion or exclusion of any particular cohort in the sample.

[19] While it it has not been central to the previous discussion in the broader diagnostic expectations literature, the result of $\theta < 0$ is not necessarily a rare sight. For instance, Bordalo et al. (2020) find $\theta < 0$ for the forecasts of a subset of variables in the Survey of Professional Forecasters. For a further discussion see Liao (2023).

dalo et al. (2019, 2020), current news are processed with a Kalman filter, so, in a sense, current forecasts are partly processed with past information. With that modeling approach, surprises with respect to a reference based on the previous period also come from a reference based on the available individual experience, as suggested by Tversky & Kahneman (1973).

However, this result changes when we consider other forms for current expectations instead of a Kalman filter, such as rational expectations (for instance, Bianchi et al. 2021 and L'Huillier et al. 2021). In that case, agents know the model's features, so their reference is based on everything contained within the model, which is considered more likely (as in Kahneman & Tversky 1972). Given the assumption of full information, agents will overreact to any shock that was not expected in the previous period; as agents understand the shock and its effects as soon as they see it, it is immediately more likely.[20]

We take a different approach, as we explicitly model the reference of the agents given their cohort. Agents also optimally generate a forecast with a Kalman filter and a signal we provide. Then, they combine this optimal forecast with their past experience to produce their final forecasts. Our framework allows for a common component that might come from the news (Carroll, 2003), common price experiences (D'Acunto et al., 2021), among other reasons. While the time fixed effect would capture any of those factors, we model it with a common adjustment to their price experience. Then, we explicitly model the reference given their history. Our findings are consistent with consumers using an availability heuristic to form expectations. In this case people use their history of forecasts to positively weigh them given the current developments in prices. When events deviate from what an agent has experienced, they tend to revise their expectation toward their historical forecasts. Our empirical results strongly point in that direction. This finding is consistent with consumers' survey-based evidence that shows that consumers, after receiving news about the economy, do not overweigh those news, keeping a positive weight on their prior or reference, as shown, for example, in Weber et al. (2023).

Using our estimate for the parameter, we perform a forecasting exercise using the Kalman filter. Panel (c) of Figure 4 shows the 12-months-ahead inflation forecasts for different cohorts. We can see that the history-based Kalman filter can generate a heterogeneous pattern across cohorts,

[20]See Benjamin (2019) for a discussion of these assumptions

similar to the one we saw in the data in Figure 1.[21] First, older cohorts show higher inflation expectations than the rest of the cohorts throughout most of the sample, based on their experiences of high inflation in the 60s, 70s, and early 80s. Second, the intermediate cohorts show lower inflation expectations than the other cohorts because they went through the stable and low inflation rates of the 90s, 00s, and 10s. The youngest cohort shows the highest inflation expectations after being exposed to the high inflation rates of 2021.

To study the external validity of our results, in Appendix G, we repeat the exercises but using data from the Consumer Expectations Survey of the European Central Bank, which contains observations for six countries: Belgium, France, Germany, Italy, the Netherlands, and Spain. We find evidence that corroborates our main findings. The inflation expectations are also heterogeneous in Europe and partially explained by the history of consumers' inflation experiences. We find support for the use of a history-based Kalman filter as a way of modeling heterogeneous inflation expectations.

Moreover, given the cross-country panel structure, we control for a common cohort fixed effect in this exercise, as in Hajdini et al. (2022a). This is a highly relevant step since common cohort characteristics (i.e. different patterns of inflation exposure over the life cycle) could affect our results. By controlling for cohort fixed effects, we rule out those common cohort characteristics and cleanly exploit differences in the inflation experienced by the different cohorts across different countries. We find similar results after adding our set of controls, implying that the heterogeneity does not stem from common cohort characteristics but from different inflation experiences in different countries. Our findings are also valid for Europe.

#### IV.2.3 Goodness of fit

This subsection studies how the time and individual variation of our history-based Kalman filter expectations compare to the data. We run a regression where the dependent variable is the survey individual inflation expectation, while the right-hand side variable is our history-based Kalman filter expectation. Our formulation has two components: one comes from the standard Kalman filter that uses common food inflation data as a signal, and the second arises from cohorts references and a parameter $\theta$ estimated using survey data. As we use a time fixed effect for the parameter

[21] We show the forecasts for the full sample in Figure A1 in Appendix A.

estimation, individual survey data does not provide the time variation observed in our inflation expectations measure. Column 1 of Table 3 shows that the slope between the survey forecast and the forecast produced by our history-based Kalman filter is 0.888. This result confirms that our inflation expectations formulation effectively reproduces consumers' expectations. After considering the time and cross-section variation, our estimates provide a good prediction of heterogeneous inflation expectations.

Table 3: Goodness of fit

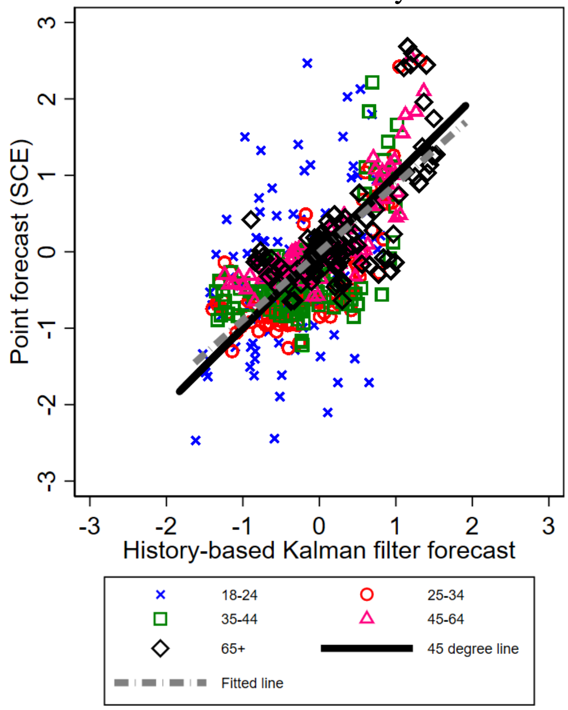
| | (1) | (2) | (3) |
|---|---|---|---|
| $\mathbb{E}^{\theta}_{i,t}\left[\pi_{t+12}\right]$ | 0.888*** | | 0.878*** |
| | (0.037) | | (0.038) |
| $\overline{\pi_{i,t}}$ | | 0.224*** | 0.029 |
| | | (0.027) | (0.029) |
| Observations | 101,262 | 101,262 | 101,262 |
| R-squared | 0.036 | 0.004 | 0.036 |

**Note**: Table shows results of a regression where the dependent variable is consumers' inflation expectations according to the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York. $\mathbb{E}^{\theta}_{i,t}\left[\pi_{t+12}\right]$ is our estimated measure of inflation expectations. $\overline{\pi_{i,t}}$ is average inflation expectations. Standard errors clustered at the date-of-birth level in parentheses. Dependent variable trimmed at 10 percent and 90 percent in each period. *** p<0.01, ** p<0.05, * p<0.1.

An alternative good measure to explain the heterogeneity in survey data inflation expectations is the cohort's lifetime average inflation rate $\overline{\pi_{i,t}} = \left(t - k_i + 1\right)^{-1} \sum_{j=0}^{k_i} \pi_{t-j}$. Column 2 of Table 3 shows that the history of inflation by cohort can also predict part of the variation in the data. By estimating a regression with both variables, we can make the history-based Kalman filter compete with the lifetime average inflation rate and see which measure better predicts the forecasts observed in the data. Column 3 of Table 3 shows that in a horserace, our model forecast is superior to the lifetime average inflation rate for explaining the observed heterogeneity. The coefficient for our measure is close to one and statistically significant. In contrast, the coefficient for the history of inflation by cohort approaches zero in magnitude and becomes statistically insignificant.

Figure 5 visually presents the results of Column 1 in Table 3. The slope associated with the regression and a 45-degree line are very close. Our measure effectively models the time and cross-sectional variation of consumers' inflation expectations.

Figure 5: Observed inflation forecasts and history-based Kalman filter forecasts




**Note**: Figure shows binned scatterplot across history-based Kalman filter forecasts (x-axis) and point forecast inflation expectations according to the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York (y-axis). Variables demeaned by the intercept. Data go from June 2013 to December 2021. SCE variable trimmed at 10 percent and 90 percent in each period.

Overall, our measure shows a very good fit with the data. We can replicate heterogeneous inflation expectations at the individual level using a relatively simple model of expectations. We can effectively replicate the time and cross-sectional variation on individual survey data inflation expectations by constructing a simple model based on current inflation and heterogeneous histories.

# V Aggregate implications of heterogeneous expectations

This section presents an overlapping generations monetary model based on an expectation formation process that replicates the heterogeneity observed in inflation expectations data (i.e., Figure 1). We assume that agents follow the history-based Kalman filter and the assumptions introduced in Section IV.2. The long history of references inherent to this approach allows for different past inflation experiences to shape different current inflation expectations across cohorts.

## V.1 Households

On the demand side, we assume that an infinite number of cohorts populates the economy. Every cohort comprises a continuum of households whose situation is summarized by a single cohort-specific representative agent. These cohorts are heterogeneous in their date of birth and past inflation experiences. We follow the perpetual youth approach presented in Blanchard (1985) and Yaari (1965) to model the different cohorts. This approach considers that households have uncertainty regarding the date on which they will die. They only know they face a mortality rate of $\lambda \in (0,1)$ every period. At the same time, a new cohort of size $\lambda$ is born every period. Therefore, in a given period $t$, the size of a cohort born in period $k \leq t$ corresponds to $\lambda\left(1-\lambda\right)^{t-k}$.

We assume households form their expectations using the history-based Kalman filter presented in Equation 3, and the assumptions from Section IV.2 hold. Agents assume that the output gap and the inflation variable are random walk processes. As in Gutiérrez-Daza (2022), agents cannot directly observe these variables; instead, they observe signals on each of these.[22] Under these assumptions, the representative household of cohort $i$ solves

$$\max_{\left\{C_{i,j},L_{i,j},B_{i,j+1}\right\}_{j\geq t}} \left(\frac{C_{i,t}^{1-\sigma}}{1-\sigma}-\frac{L_{i,t}^{1+\eta}}{1+\eta}\right)+\mathbb{E}_{i,t}^{\theta}\left[\sum_{j>t}\left[\beta\left(1-\lambda\right)\right]^{j-t}\left(\frac{C_{i,j}^{1-\sigma}}{1-\sigma}-\frac{L_{i,j}^{1+\eta}}{1+\eta}\right)\right],$$

subject to

$$P_tC_{i,t}+\left(1-\lambda\right)\frac{B_{i,t+1}}{\left(1+i_t\right)}=W_tL_{i,t}+B_{i,t}+T_{i,t},$$

where $C_{i,t}$ is consumption, $L_{i,t}$ is the labor supply, $B_{i,t}$ are nominal savings in a risk-less bond, $P_t$ is the aggregate price level, $W_t$ are the nominal wages, $T_{i,t}$ are transfers, and $i_t$ is the nominal interest rate associated with this economy's bond. Also, $\beta$ is the discount factor, $\sigma$ is the intertemporal elasticity of substitution, and $\eta$ is the inverse of the Frisch elasticity.

The transfers $T_{i,t}$ are crucial to our model, as they incorporate two different mechanisms. First, as in Blanchard (1985) and Yaari (1965), we assume that households insure themselves to receive a flow of income every period they are alive. Then, when they die, the insurance company takes away any residual wealth. Thus, there are no accidental bequests. Second, as in Mankiw & Reis (2006),

[22] For simplicity, we assume that these signals correspond to the lagged value of each of the forecasted variables.

we assume that the income households receive each period from the insurance company is such that households start each period with the same wealth and that the nominal savings market clears. Therefore, we do not have to worry about wealth distribution. Lastly, and as a way of closing the model, the transfers also incorporate the benefits of firms producing intermediate goods.

Additionally, the transversality condition of the problem is

$$\lim_{T\to\infty} \frac{(1-\lambda)^T}{\Pi_{h=0}^{T-1}(1+i_{t+h})} B_{i,t+T} = 0.$$

We introduce expectations in a general equilibrium setting as in Bianchi et al. (2021) and L'Huillier et al. (2021). The expectations operator used by households is $\mathbb{E}_{i,t}^{\theta}[.]$, which works under the assumptions made in Section IV.2. Therefore, the first-order conditions of this problem are

$$\left(P_t C_{i,t}^{\sigma}\right)^{-1} = \beta(1+i_t)\,\mathbb{E}_{i,t}^{\theta}\left[\left(P_{t+1}C_{i,t+1}^{\sigma}\right)^{-1}\right], \tag{7}$$

$$C_{i,t}^{\sigma} L_{i,t}^{\eta} = \frac{W_t}{P_t}, \tag{8}$$

where the first equation corresponds to our Euler equation, and the second denotes a standard labor supply condition.

To simplify the problem, we further assume that the agents know the current state of the economy, and for any log-linearized variable $x_t$ we have $\mathbb{E}_{i,t}^{\theta}[x_{t-h}] = x_{t-h}$ when $h \geq 0$. Otherwise, as L'Huillier et al. (2021) explains, agents will over or underreact when predicting this variable's present or past values. The log-linearization of Equation 7, following the history-based Kalman filter presented in Equation 3, results in

$$c_{i,t} = \begin{array}{c} (1+\theta)\,\mathbb{E}_t^{KF}[c_{i,t+1}] - \frac{1}{\sigma}\left(i_t - (1+\theta)\,\mathbb{E}_t^{KF}[\pi_{t+1}]\right) \\ -\theta\left(\mathbb{E}_{i,t}^{ref}[c_{i,t+1}] + \dfrac{1}{\sigma}\mathbb{E}_{i,t}^{ref}[\pi_{t+1}]\right), \end{array} \tag{9}$$

where lower cases denote deviations from the steady state.[23] This is the IS curve associated with

[23]An intermediate step in the log-linearization of Equation 7 results in $c_{i,t} = \mathbb{E}_{i,t}^{\theta}[c_{i,t+1}] - \frac{1}{\sigma}\left(i_t - \mathbb{E}_{i,t}^{\theta}[\pi_{t+1}]\right) + \frac{1}{\sigma}\sum_{j=0}^{k_i+1}\left(\mathbb{E}_{i,t}^{\theta}[\pi_{t-j}] - \pi_{t-j}\right)$, where the last term results from the fact that $\mathbb{E}_{i,t}^{\theta}[X_{t+1}Z_t] \neq \mathbb{E}_{i,t}^{\theta}[X_{t+1}]\,Z_t$ (L'Huillier et al., 2021). The term $\mathbb{E}_{i,t}^{\theta}[\pi_{t-j}]$ implies that agents also have distorted beliefs about the past. We assume $\mathbb{E}_{i,t}^{\theta}[\pi_{t-j}] = \pi_{t-j}$

a given cohort $i$.

### V.1.1 Aggregation

Since the steady-state value is the same across the different cohorts, the aggregate consumption gap $c_t$ corresponds to the weighted sum of all the different cohort-level consumption gaps. Therefore, we establish

$$c_t = \lambda \sum_{k\geq 0} (1-\lambda)^k c_{k,t}. \tag{10}$$

We further assume that household $k$, when forecasting its future individual consumption gap, believes that all the other households will behave similarly such that $\mathbb{E}_{k,t}^{ref}\left[c_{k,t+1}\right] = \mathbb{E}_{k,t}^{ref}\left[c_{t+1}\right]$. Incorporating Equation 9 into Equation 10 and using the market clearing condition in the goods market, we conclude:

$$y_t = \begin{array}{l} (1+\theta)\,\mathbb{E}_t^{KF}\left[y_{t+1}\right] - \frac{1}{\sigma}\left(i_t - (1+\theta)\,\mathbb{E}_t^{KF}\left[\pi_{t+1}\right]\right) \\ -\theta\lambda \sum\limits_{k\geq 0} (1-\lambda)^k \left(\mathbb{E}_{k,t}^{ref}\left[y_{t+1}\right] + \dfrac{1}{\sigma}\mathbb{E}_{k,t}^{ref}\left[\pi_{t+1}\right]\right). \end{array} \tag{11}$$

Equation 11 is the IS curve associated with our model. It equals the standard IS curve plus two different distortions associated with parameter $\theta$. In this version of the IS curve, the past matters in the sense that current realizations are affected by the history of inflation of the cohorts.[24]

## V.2 Firms

On the supply side, we assume that a final good producer operates in a perfectly competitive market. This firm produces using a continuum of intermediate goods as inputs. Therefore, there is a continuum of intermediate goods producers, each one operating under monopolistic competition. These intermediate goods producers are subject to Calvo pricing frictions. We assume these firms follow rational expectations when setting their prices because they are model-consistent. Thus,

for $j \geq 0$. That is, the beliefs about the past align with the observed variables. We follow this assumption because we focus on how the reference distortions affect beliefs about the future, not the past. Importantly, this assumption also makes the solution of the model computationally simpler, as we do not have to keep track of the beliefs of each cohort for each past period. Therefore, we drop the last term and obtain Equation 9.

[24] Notice that if agents do not over or underreact when forming expectations, i.e. $\theta = 0$, the IS curve reduces to $y_t = \mathbb{E}_t^{KF}\left[y_{t+1}\right] - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t^{KF}\left[\pi_{t+1}\right]\right)$.

we adopt the standard firms' framework associated with the New Keynesian setting to obtain the standard New Keynesian Phillips curve.[25]

### V.3 Monetary policy

The central bank sets the interest rate following a standard Taylor rule. Then, we have

$$\frac{(1+i_t)}{(1+\bar{i})} = \left[\frac{(1+\pi_t)}{(1+\bar{\pi})}\right]^{\chi_\pi} \left[\frac{Y_t}{\bar{Y}}\right]^{\chi_y}, \tag{12}$$

where the bars denote steady state values and $\chi_\pi$ and $\chi_y$ represent the central bank's reaction to deviations from the steady state of the inflation rate and output, respectively.

After log-linearizing, the model is summarized by

$$y_t = \begin{array}{c} (1+\theta)\mathbb{E}_t^{KF}[y_{t+1}] - \frac{1}{\sigma}\left(i_t - (1+\theta)\mathbb{E}_t^{KF}[\pi_{t+1}]\right) \\ -\theta\lambda \sum_{k\geq 0}(1-\lambda)^k \left(\mathbb{E}_{k,t}^{ref}[y_{t+1}] + \frac{1}{\sigma}\mathbb{E}_{k,t}^{ref}[\pi_{t+1}]\right) + u_t^{taste}, \end{array} \tag{13}$$

$$\pi_t = \frac{(1-\phi)(1-\phi\beta)}{\phi}(\sigma+\eta)\left(y_t + u_t^{cost}\right) + \beta\mathbb{E}_t[\pi_{t+1}], \tag{14}$$

$$i_t = \chi_\pi \pi_t + \chi_y y_t, \tag{15}$$

where Equation 13 is the dynamic IS curve augmented with a taste shock, Equation 14 is the Phillips curve, and Equation 15 is the Taylor rule. Notice the Phillips curve follows the rational expectations operator $E_t[.]$, while the IS curve results from following the history-based Kalman filter operator $E_t^\theta[.]$.

We consider a cost shock $u_t^{cost}$ and a taste shock $u_t^{taste}$ that behave as an AR(1) process, such that

$$u_t^{cost} = \rho_{cost} u_{t-1}^{cost} + \varepsilon_t^{cost}, \tag{16}$$

$$u_t^{taste} = \rho_{taste} u_{t-1}^{taste} + \varepsilon_t^{taste}, \tag{17}$$

[25]We present the details associated with the supply side of the economy in Appendix H.

where $\rho_{cost}$ and $\rho_{taste}$ are the persistence parameter and $\varepsilon_t^{cost}$ and $\varepsilon_t^{taste}$ are the unexpected innovations.

## V.4 Calibration

The model is calibrated to a monthly frequency. The parameters from Table A1 in Appendix A show a fairly standard calibration. We calibrate the price stickiness parameter $\phi$ so that the expected duration of a given price quote is 12 months. We also calibrate the mortality rate $\lambda$ so that the expected life span is 80 years.[26]

Regarding the history-based Kalman filter, we need to calibrate the Kalman gain $K$ and the parameter $\theta$. We calibrate both according to the results from Sections IV.1 and IV.2. We must make an additional assumption around these two parameters. While we only used inflation rate data in the previous sections, we assume these parameters also hold for the output gap.

## V.5 Simulations

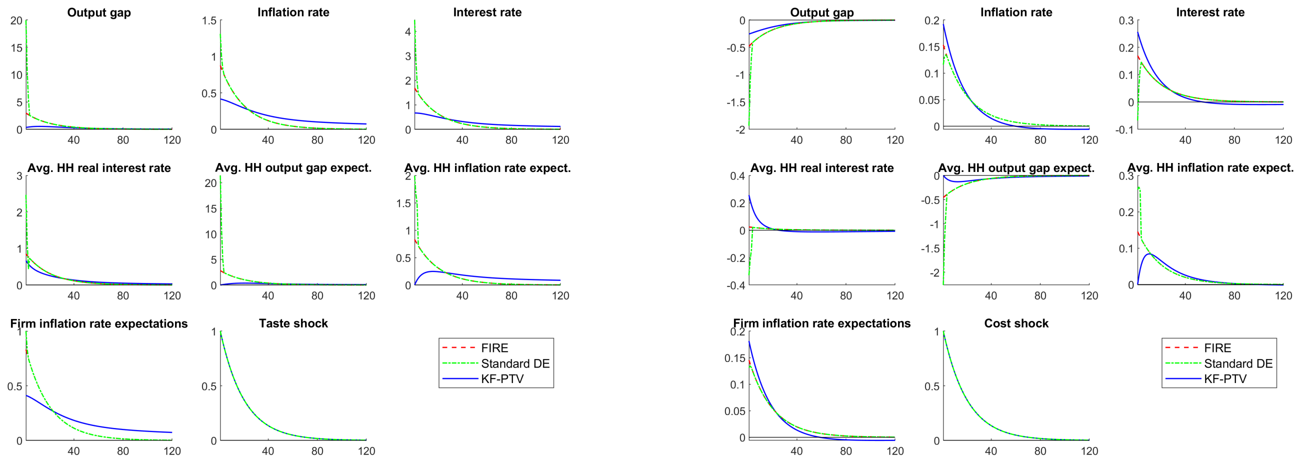
In this section we compare three different cases: (i) households form their expectations according to full information rational expectations (FIRE), (ii) households form their expectations according to a standard diagnostic expectations operator with overreaction,[27] and (iii) households form their expectations according to the history-based Kalman filter from Equation 3.[28]

[26]Because we assume that agents become economically active and relevant at age 18, this means agents expect to consume and work for 62 years

[27]We include the standard diagnostic expectations model because it serves as an useful benchmark that features overreaction. Because our model is a generalization of the diagnostic expectations model, following L'Huillier et al. (2021), we set the parameter $\theta = 0.992$ to obtain the desired overreaction result.

[28]For our Kalman filter operator we assume each cohort differs in its beliefs. In Appendix I we analyze the consequences of dropping this assumption and having all cohorts follow the same beliefs while keeping the Kalman filter structure.

Figure 6: Impulse response functions




(a) Taste shock (b) Cost shock

**Note**: Figure shows impulse response functions for a selected group of variables after the mentioned shocks. The red dashed line shows the results for the case of the full information rational expectations model (FIRE), the green dotted line shows the results of a standard diagnostic expectations operator and the solid blue line shows the history-based Kalman filter model. For the standard diagnostic expectations case we assume that agents use the expectations operator $\mathbb{E}_t^{\theta,s}[X_{t+h}] = \mathbb{E}_t[X_{t+h}] + \varpi(\mathbb{E}_t[X_{t+h}] - \mathbb{E}_{t-3}[X_{t+h}])$ with $\varpi > 0$. Horizontal axis denotes months after the shock.

Figure 6 presents the impulse response functions to a taste shock and a cost shock. In Panel (a), we see that inflation and the output gap increase after a taste shock. The FIRE case shows the usual reaction, and the standard diagnostic expectations model presents an overreaction that lasts three periods, consistent with the reference used. In the case of the history-based Kalman filter model, we first see a milder reaction in terms of inflation and output gap. Agents remember the past (in this case, the steady state), so their expectations tend to stay closer to such value. This allows the central bank to reduce the size of its hike in the interest rate, resulting in a slightly lower real interest rate. In addition, firms reduce the size of their price increase. Overall, under the history-based Kalman filter, agents anchor their expectations to the past, reducing the magnitude of the responses on impact.

While inflation is lower on impact with the history-based Kalman filter, it takes longer to return to its steady state values when compared to the other cases (the FIRE and standard diagnostic expectations cases have inflation going back to a steady state at the same pace of the persistence of the shock). This is because, with the history-based Kalman filter, agents remember the high inflation period. This effect is exacerbated by new cohorts that have only experienced inflation above the steady state.

Panel (b) of Figure 6 shows the responses for a cost shock. In the history-based Kalman filter case, there are two forces going in opposite directions. There is a high persistence in inflation, but we also have consumers who remember the zero output gap of the steady state, which reduces the pressure on prices. Thus, the IS curve becomes more inelastic to the shock with the history-based Kalman filter. Then, in this economy, rational firms can raise prices by more than they would under FIRE. This is followed by a central bank that must raise the interest rate more strongly than in the rational economy.

One thing to notice in the history-based Kalman filter case is that while the household inflation rate expectations have a hump shape, actual inflation does not. This is because firms are always rational. Thus, their expectations follow the shock very closely (no hump shape because of the AR(1) nature of the shock), and they set prices accordingly. The hump-shaped expectations are consistent with the evidence provided by Angeletos et al. (2021). Consumers underreact to the inflationary shock in our setting initially, as their memories are tied to the steady state. After the inflationary shock, they incorporate the inflationary episode in their memories, over-extrapolating it, as in Afrouzi et al. (2023).

## V.6 Implied heterogeneous responses

After obtaining the aggregate responses to the shocks, we now check the responses at the cohort-specific level implied by the aggregate variable paths of the previous section. Thus, we use the individual level equations to see how the informational frictions of the model generate heterogeneous consumption and labor patterns depending on the past experiences of the cohorts. We must stress this result just shows a reaction starting akin to a partial equilibrium exercise taking the aggregate variable paths as given and is not the result of full heterogeneous agents model.

First, we calculate the within-model response of the output gap and inflation rate expectations for each cohort according to the framework of Section IV.2 and the aggregate variable paths following the shocks studied in the previous section. Figures A3 and A4 of Appendix A present the heterogeneity in expectations across cohorts under the history-based Kalman filter. We see that, after both shocks, the expectations of the older cohorts remain anchored to the steady state, whereas the young react more strongly.

Then, we obtain cohort-specific consumption gap according to Equation 9, which contains common and heterogeneous components across cohorts. Similarly, we obtain the cohort-specific paths of labor supply gap from the individual labor supply problem. Figure A5 in Appendix A shows the consumption and labor supply gap responses of the different cohorts after a taste shock. We see that older cohorts consume less and work more, relative to the younger ones. For this result, there are two forces working simultaneously. In first place, the output gap expectations are lower for older cohorts, which brings their consumption down. In second place, the inflation rate expectations are lower for older cohorts, which relates to a higher real interest rate and reinforces the lower consumption. Because the older cohorts believe the future aggregate output gap will be lower and perceive a higher real interest rate, they end up supplying more labor relative to the young.

Figure A6 of Appendix A shows the responses of the consumption and labor supply gap across cohorts after a cost shock. Older cohorts consume more and supply less labor than the young. Contrary to the previous exercise, in this case this outcome is explained by two channels going in opposite directions. First, older cohorts have higher output gap expectations than than younger ones, which drives their consumption upwards. Second, older cohorts have lower inflation rate expectations that translate into higher real interest rates, such that their consumption goes down. In the end, we see that the first channel dominates over the second. That is, even though older cohorts perceive a higher real interest rate, their more optimistic beliefs over the future output gap drive their consumption upwards relative to the young cohorts.

We highlight that these results are specific to this exercise, where the economy starts in the steady state. The heterogeneous responses are determined by the heterogeneous experiences of households which, given the history-based biases when forming expectations, create heterogeneous ex-ante real interest rates. In Section VII we explore a data-based inflation scenario that will result in different ex-ante real interest rates across cohorts.

# VI Optimal Taylor rules

Now we analyze the use of an optimal Taylor rule in each of the different cases from the previous section. The Taylor rule we use in this section is

$$i_t = \chi_\pi^* \pi_t + \chi_y^* y_t. \tag{18}$$

We assume that the central bank chooses the time-invariant parameters $\chi_\pi^*$ and $\chi_y^*$ such that it solves the problem

$$\min \mathbb{E}_t \left[\pi_t^2 + \vartheta y_t^2\right],$$

subject to the equations of the model in Section V and $\vartheta$ is the weight of the output gap in the objective function.[29] That is, the central bank, given the model setup, seeks to minimize the volatility of both the inflation rate and the output gap. Notice that we assume that the central bank has rational expectations.

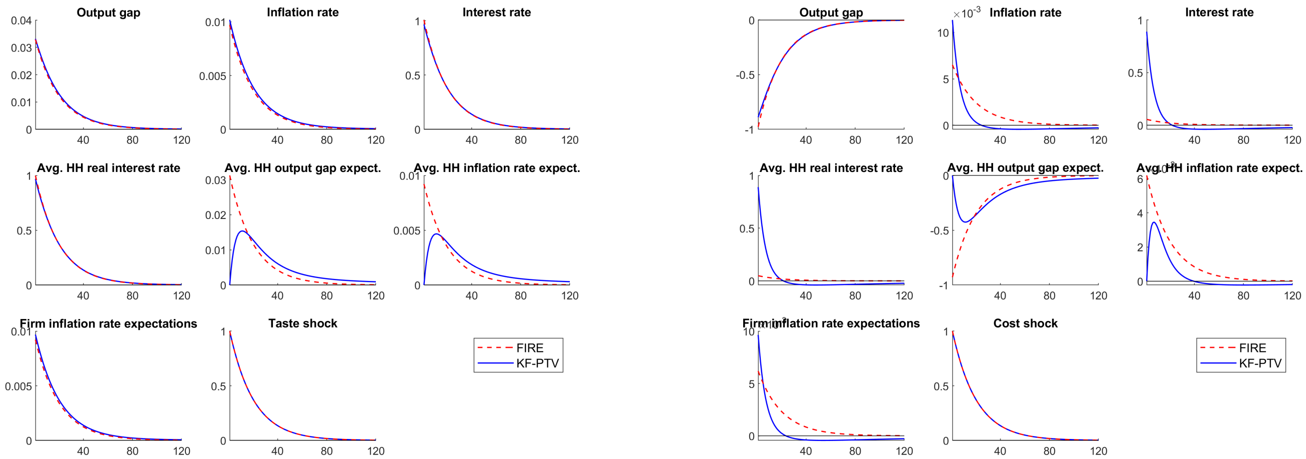
The optimal parameters are dependent on which shocks exist in the model (cost or taste). Therefore, we will have two sets of parameters, one for each shock.[30]

We start by analyzing the response to a cost shock under the optimal Taylor rule. Panel (b) of Figure 7 shows that, when responding to this shock, the central bank faces the usual trade-off between the output gap and the inflation rate. After the cost shock, the inflation rate goes up and the output gap goes down after the interest rate hikes. In this particular exercise we will have that, given the relative importance of the output gap in the objective function, the central bank will favor reducing the volatility of the inflation rate.

[29] Following Gali (2015) we define $\vartheta = \frac{(1-\phi)(1-\phi\beta)(\sigma+\eta)}{\phi\varepsilon} = 0.0017$.

[30] Tables A2 and A3 in Appendix A show the optimal parameters.

Figure 7: Impulse response functions, optimal Taylor rule




(a) Taste shock (b) Cost shock

**Note**: Figure shows impulse response functions for a selected group of variables after the mentioned shocks under an optimal Taylor rule. The red dashed line shows the results for the case of the full information rational expectations model (FIRE) and the solid blue line shows the history-based Kalman filter model. Horizontal axis denotes months after the shock.

After the shock hits, the central bank becomes more active in the history-based Kalman filter than in the FIRE case. This behavior is because inflation history plays a role when there are history-based expectations. The central bank knows that people will remember the current shock far into the future, affecting future inflation expectations. By being more active under the history-based Kalman filter case, the central bank can quickly lower inflation expectations. While in the baseline results from Figure 6 the inflation expectations remained high for a long period, the optimal Taylor rule brings them down. It even generates deflation expectations that later spill over to the observed inflation rate.

Panel (a) of Figure 7 shows the impulse response functions to a taste shock and an optimal response from the central bank. As it is well known in the literature, upon a taste shock, the optimal response of the central bank is to raise the interest rate strongly. What follows is that the central bank manages to bring down both the output gap and the inflation rate to their steady state values.

We find no significant difference in the central bank's response between the FIRE and history-based cases. The optimal Taylor rule case says that the central bank should always be active when facing a taste shock, no matter the type of expectations agents have. In this manner, the central bank can close the output gap and lower the inflation rate faster than the baseline results.

Moreover, with the active stance recommended by the optimal Taylor rule, the output gap and inflation expectations are positive but very close to zero in all cases.

# VII Analyzing an episode of high inflation

This section analyzes the model's behavior after the high-inflation episode of 2021.[31] To do so, we feed the model from Section V with actual monthly data on the output gap, inflation rates, and interest rates up to December 2021.[32] Afterward, we produce forecasts using the different versions of the model.[33]

We first analyze the history-based Kalman filter case. Figure A7 of Appendix A shows the inflation rate expectations by cohort according to our model and the data. Before 2021, older cohorts had the highest inflation expectations. This result occurs because older cohorts experienced the high-inflation episodes of the 60s, 70s, and early 80s. They are followed by the intermediate cohorts and finally by the youngest cohorts, who experienced low and stable inflation rates from the 90s to the 10s. With the inflationary shock, things changed as newer cohorts experienced a significant part of their lives in a high-inflation environment.

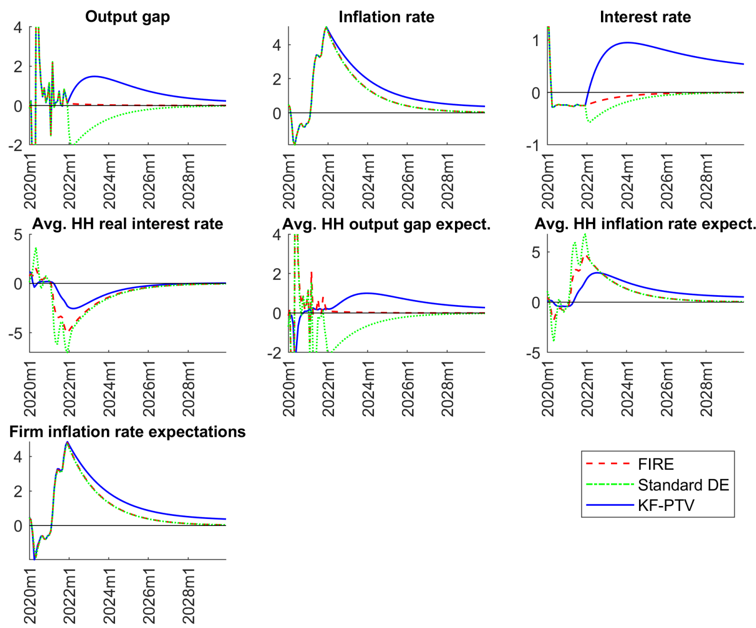
Figure 8 presents how variables evolve based on our model and the data. After 2021, when we allow for a history-based Kalman filter, average inflation expectations are higher and more persistent with respect to other cases. This result is because agents remember and anchor their expectations to what they experienced in the past. Hence, the central bank must react more strongly.

[31] In this part, we go back to the basic calibration of Table A1.

[32] We use monthly series from March 1967 to December 2021. We go as far as the data allows us to build the agents' references. We use the National Activity Index (CFNAI) from the Federal Reserve Bank of Chicago for the output gap. For the interest rate, we use the effective federal funds rate. For the inflation rate, we use the CPI 12-month percentage change.

[33] We present the shocks that, according to our model, explain the observed data in Figure A8 in Appendix A.

Figure 8: Impulse response functions, forecast




**Note**: Figure shows impulse response functions for a selected group of variables according to the model and the data (up to December 2021). The red dashed line shows the results for the case of the full information rational expectations model (FIRE), the green dotted line shows the results of a standard diagnostic expectations operator and the solid blue line shows the history-based Kalman filter model. For the standard diagnostic expectations case we assume that agents use the expectations operator $\mathbb{E}_t^{\theta,s}[X_{t+h}] = \mathbb{E}_t[X_{t+h}] + \varpi(\mathbb{E}_t[X_{t+h}] - \mathbb{E}_{t-3}[X_{t+h}])$ with $\varpi > 0$. Horizontal axis denotes months.

Because of their historical reference, agents that follow a history-based Kalman filter remember the high inflation episode far into the future when compared to the other cases case. As a consequence, the observed inflation rate and the interest rate also remain higher for longer.

# VIII Conclusions

This paper studies the macroeconomic consequences of heterogeneous inflation expectations. We first show that inflation expectations are heterogeneous across cohorts. We introduce a Kalman filter augmented with history-based expectations to model the inflation forecast formation process. We structurally estimate the relevant parameter, concluding that individuals effectively consider their past inflation histories when forecasting.

Our expectation formation process includes two known and relevant aspects of consumers' expectations processes. On one side, we consider the history of inflation each cohort has experienced. On the other side, we consider the current inflation experience, particularly with salient prices such as grocery prices. Our proposed framework can predict the heterogeneity prevalent in consumers' inflation expectations across cohorts remarkably well, even though we only feed our model with monthly CPI inflation data and an estimated parameter. We show that consumer inflation expectations data can be modeled, predicted, and contain meaningful information.

Our modeling approach also has the advantage of being flexible, such that we incorporate it in a general equilibrium model. This heterogeneous expectation process has relevant aggregate implications. Heterogeneous expectations anchor aggregate response to agents' history of inflation while increasing the persistence of the effect of the shocks.

This result has implications for monetary policy: when inflation starts rising, the optimal response of monetary authorities is to take an active stance, as agents have a long history and remember current shocks far into the future. An energetic response of the central bank under inflationary pressures prevents current inflation from rising and, more importantly, prevents agents from incorporating high-inflation episodes into their memories, thus preventing higher future inflation expectations.

Our results also have relevant implications for the current macroeconomic environment. The model suggests that the 2021 high-inflation episode, even though it may be transitory, could have long-lasting effects: new cohorts incorporate the high-inflation episode into their memories of inflation, adjusting future inflation expectations upwards.

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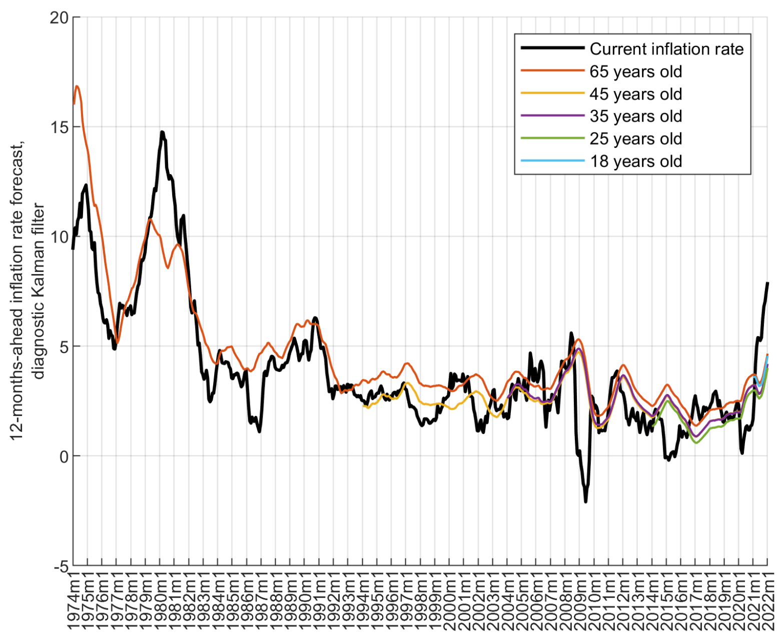
# Online Appendix (Not for publication)

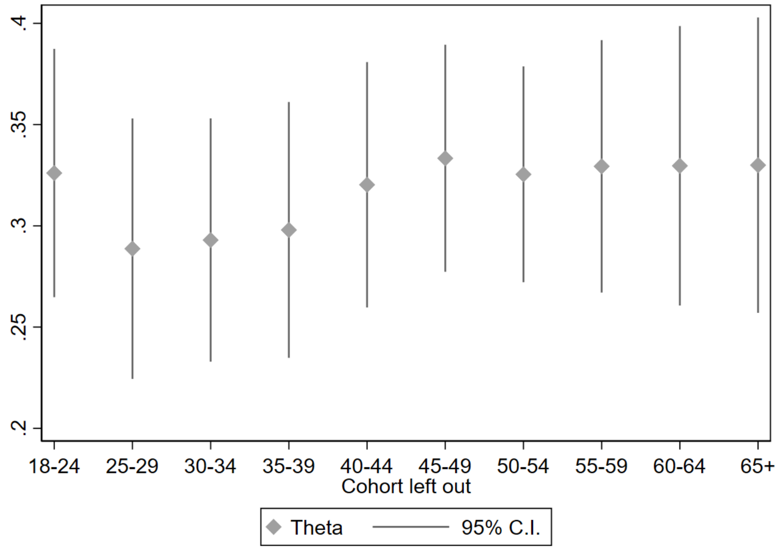
# A Additional figures and tables

Figure A1: History-based Kalman-filter-based inflation forecasts by cohort, full sample



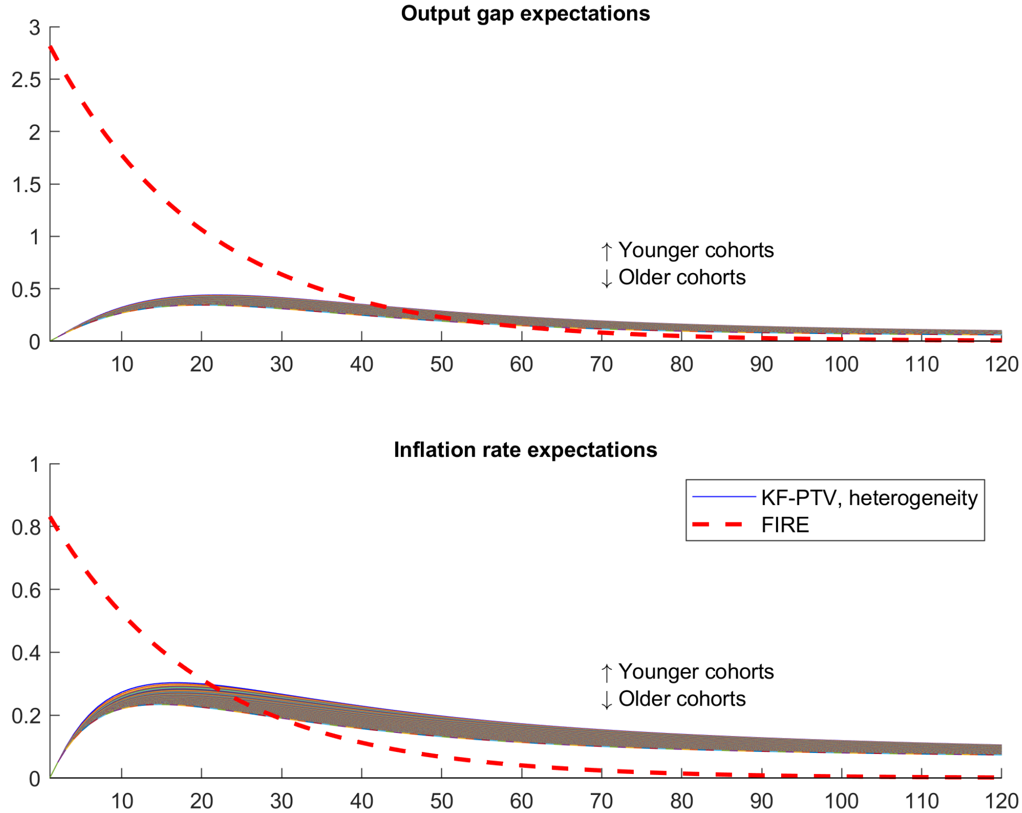

**Note**: Figure shows forecasts for selected cohorts according to the Kalman-filter-augmented expectations and considering the estimate for $\theta$ from Column 1 of Table 2. Selected cohorts differentiated by their age in 2021. We further assume that each cohort starts forecasting when they become 18 years old. The current inflation rate measure is based on the monthly YoY percentage variation of the CPI.

Figure A2: Parameter estimation, stability by cohorts



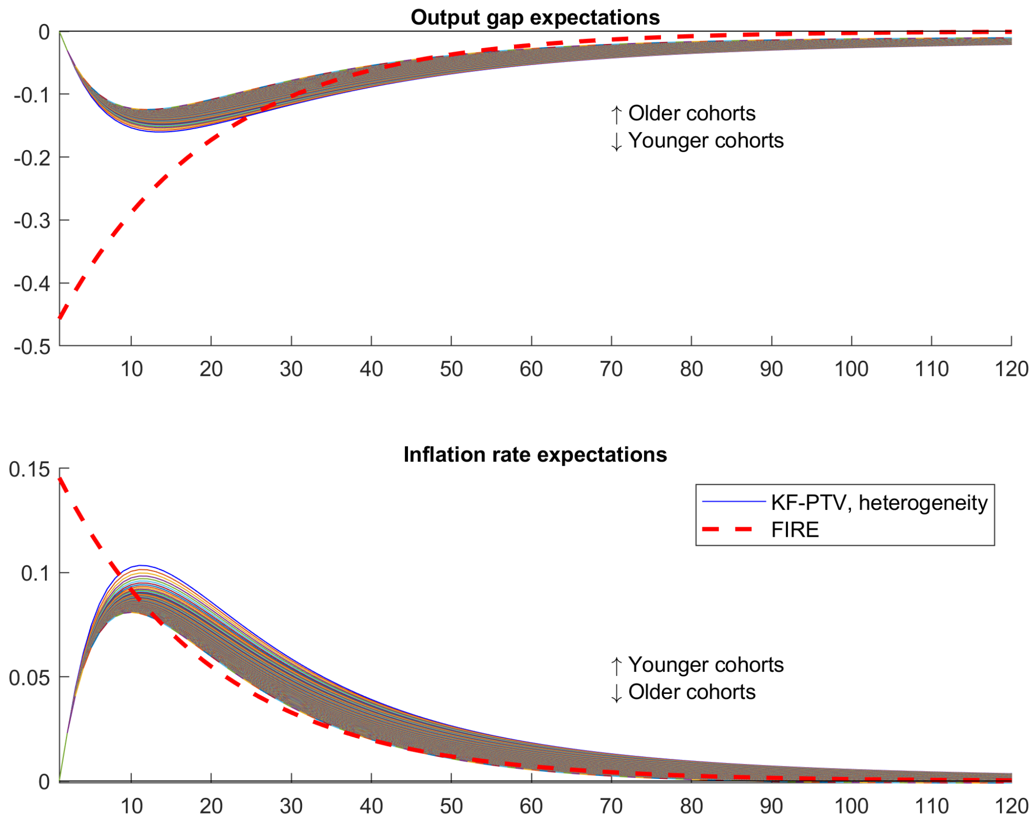

**Note**: Figure shows the coefficient resulting from estimating Equation 6, but leaving one cohort out of the sample. The cohort left out of the estimation changes across the horizontal axis.

Figure A3: Impulse response functions, history-based expectations by cohort, taste shock



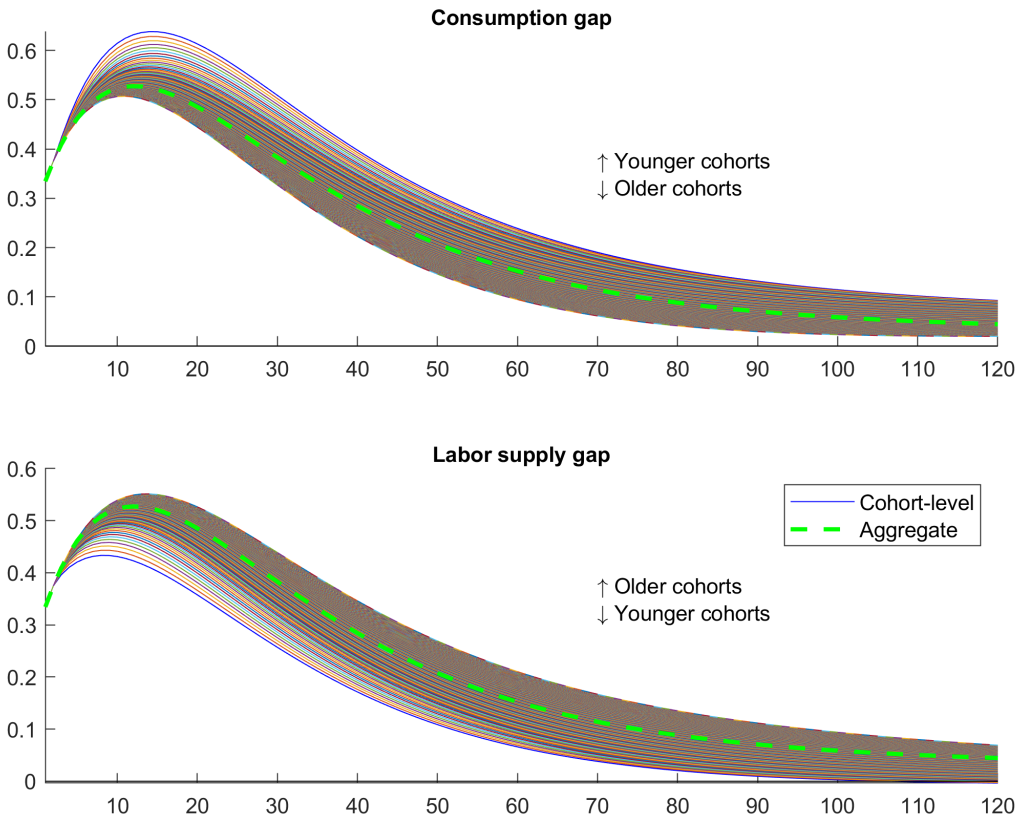

**Note**: Figure shows the heterogeneous expectations generated by the history-based Kalman filter. Cohorts denote age at the time of the shock. The solid lines represent different cohorts in the history-based Kalman filter model. The dashed red line is the full information rational expectations model. Horizontal axis denotes months after the shock.

Figure A4: Impulse response functions, history-based expectations by cohort, cost shock



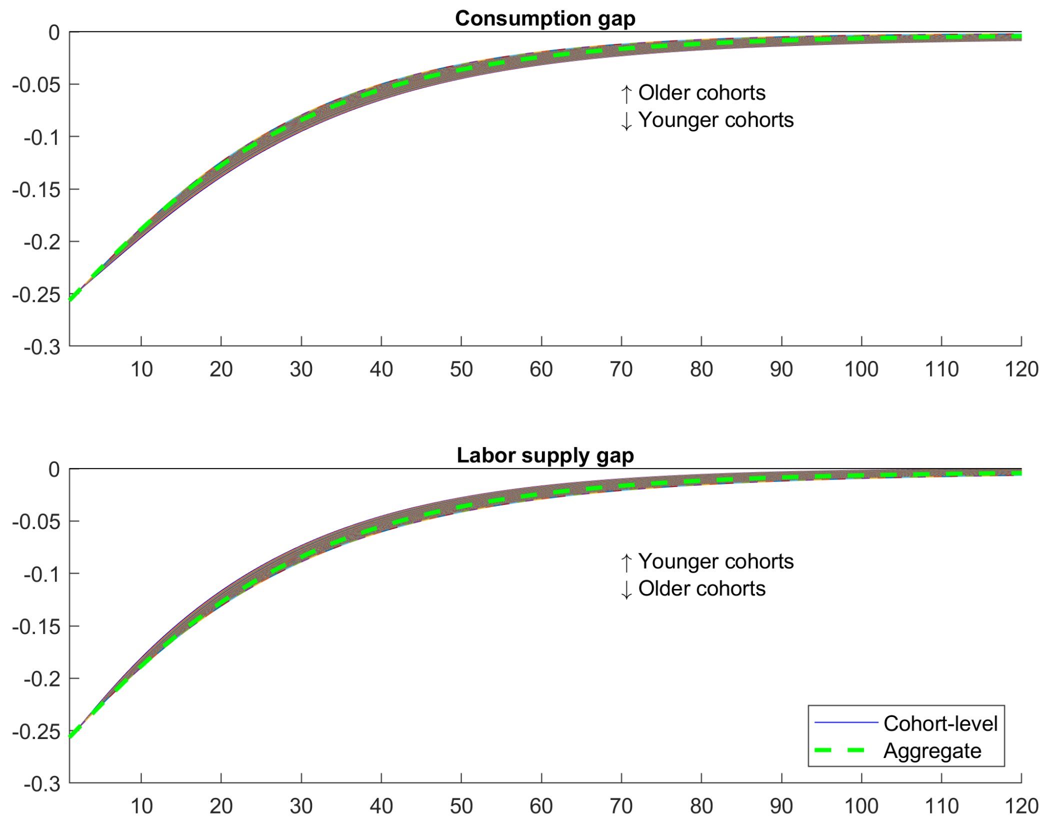

**Note**: Figure shows the heterogeneous expectations generated by the history-based Kalman filter. Cohorts denote age at the time of the shock. The solid lines represent different cohorts in the history-based Kalman filter model. The dashed red line is the full information rational expectations model. Horizontal axis denotes months after the shock.

Figure A5: Impulse response functions, consumption and labor supply by cohort, history-based KF with heterogeneity, taste shock



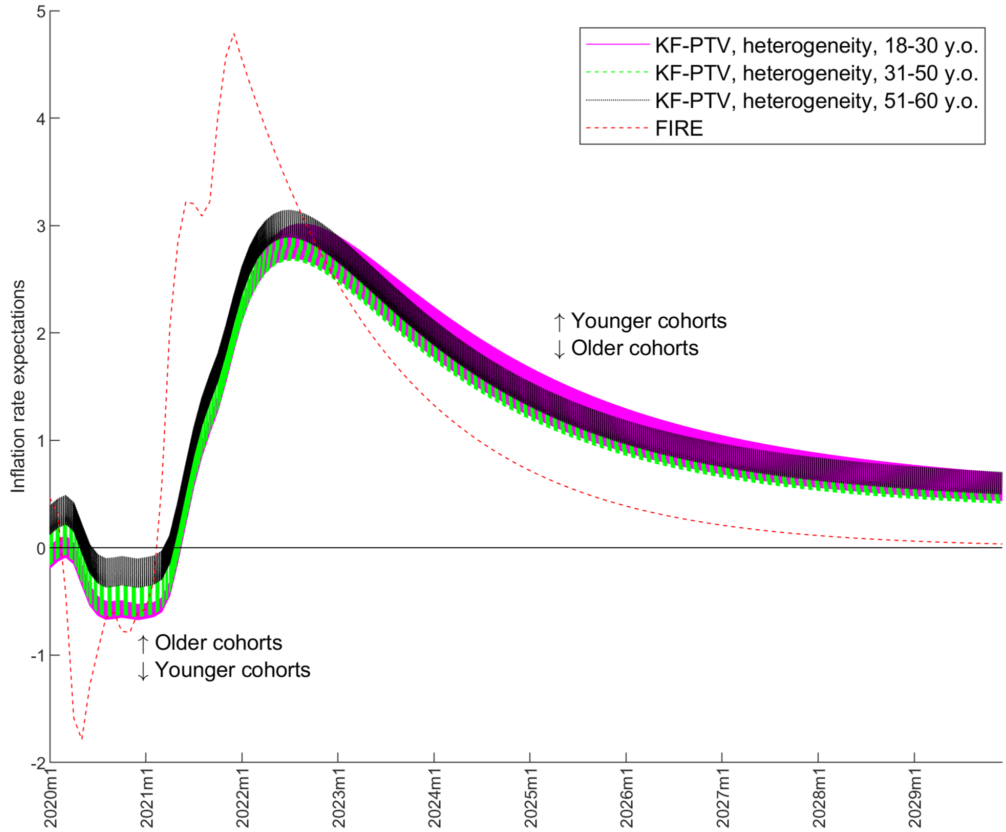

**Note**: Figure shows the heterogeneous consumption and labor supply gaps generated in the case where expectations follow the history-based Kalman filter. Cohorts denote age at the time of the shock. The solid lines represent different cohorts in the history-based Kalman filter model. The dashed red line is the aggregate. Horizontal axis denotes months after the shock.

Figure A6: Impulse response functions, consumption and labor supply by cohort, history-based KF with heterogeneity, cost shock



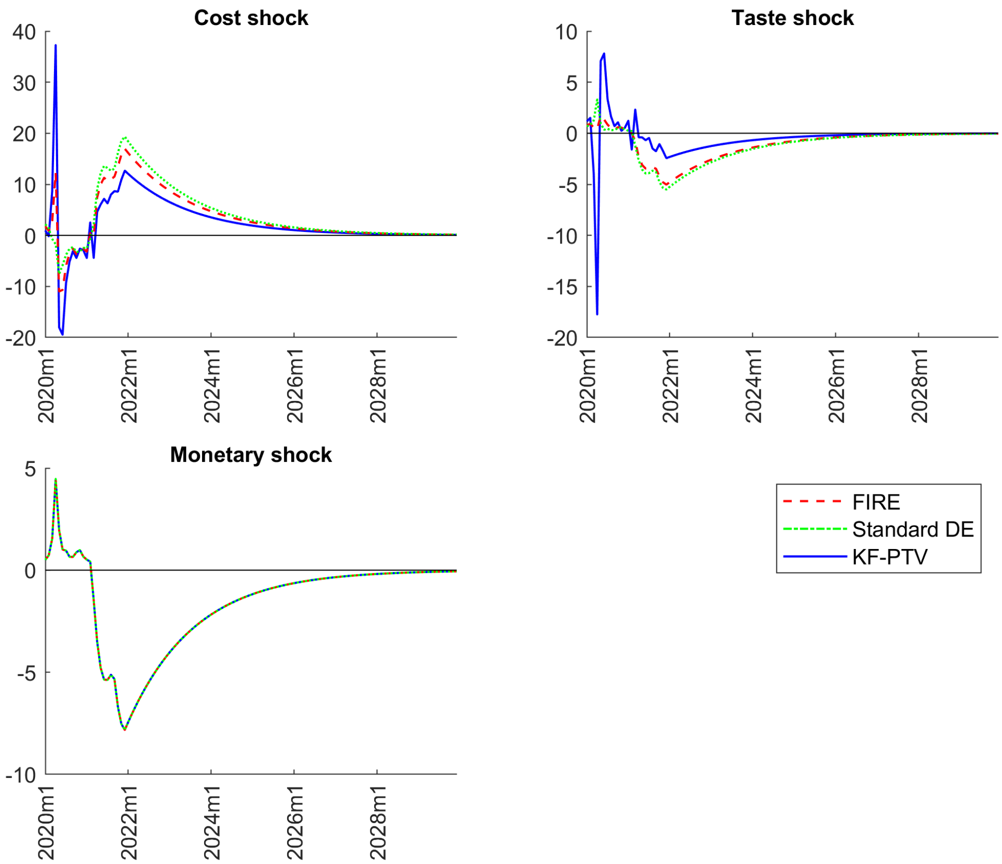

**Note**: Figure shows the heterogeneous consumption and labor supply gaps generated in the case where expectations follow the history-based Kalman filter. Cohorts denote age at the time of the shock. The solid lines represent different cohorts in the history-based Kalman filter model. The dashed red line is the aggregate. Horizontal axis denotes months after the shock.

Figure A7: Impulse response functions, inflation rate history-based expectations by cohort, forecast




**Note**: Figure shows the heterogeneous expectations generated by the history-based Kalman filter and the data (up to December 2021). Cohorts denote age in 2021. Horizontal axis denotes months.

Figure A8: Impulse response functions, shocks, forecast




**Note**: Figure shows the paths shocks follow according to the model and the data (up to December 2021). The red dashed line shows the results for the case of the full information rational expectations model (FIRE), the green dotted line shows the results of a standard diagnostic expectations operator and the solid blue line shows the history-based Kalman filter model. For the standard diagnostic expectations case we assume that agents use the expectations operator $\mathbb{E}_t^{\theta,s}[X_{t+h}] = \mathbb{E}_t[X_{t+h}] + \zeta(\mathbb{E}_t[X_{t+h}] - \mathbb{E}_{t-3}[X_{t+h}])$ with $\zeta > 0$. Horizontal axis denotes months.

Table A1: Model calibration

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $\beta$ | 0.9967 | $\chi_y$ | 0.125 |
| $\eta$ | 1 | $\rho^{cost}$ | 0.9 |
| $\phi$ | 0.9167 | $\rho^{taste}$ | 0.9 |
| $\sigma$ | 1 | $\lambda$ | 0.001 |
| $\varepsilon$ | 9 | $K$ | 0.1751 |
| $\chi_\pi$ | 1.5 | $\theta$ | -0.317 |

**Note**: Table shows the parameters used for the model. We follow a standard monthly calibration.

Table A2: Optimal Taylor rule parameters, cost shock

| | $\chi_\pi^*$ | $\chi_y^*$ |
|---|---|---|
| FIRE | 119.20 | 0.74 |
| History-based KF-PTV | 81.96 | 0.05 |

**Note**: Table shows the Taylor rule parameters that minimize objective function $\mathbb{E}_t\left[\pi_t^2 + \vartheta y_t^2\right]$ when an unexpected cost shock hits the economy.

Table A3: Optimal Taylor rule parameters, taste shock

| | $\chi_\pi^*$ | $\chi_y^*$ |
|---|---|---|
| FIRE | 8.73 | 28.10 |
| History-based KF-PTV | 9.24 | 26.37 |

**Note**: Table shows the Taylor rule parameters that minimize objective function $\mathbb{E}_t\left[\pi_t^2 + \vartheta y_t^2\right]$ when an unexpected taste shock hits the economy.

# B Normality of history-based expectations

Let $f\left(\pi_{t+1}|\mathscr{I}_t\right)$ be the true distribution of future inflation conditional on the currently available information set $\mathscr{I}_t$. We assume this behaves as

$$f\left(\pi_{t+1}|\mathscr{I}_t\right) \sim N\left(\mathbb{E}_t^{KF}\left[\pi_{t+1}\right], \sigma_\pi^2\right),$$

where $\mathbb{E}_t^{KF}\left[\pi_{t+1}\right]$ is the expectation computed according to the standard Kalman filter. On the other hand, let $f\left(\pi_{t+1}|\mathscr{I}_{i,t}^{ref}\right)$ be the distribution of the inflation rate conditional on the referential information set associated with cohort $i$. Under our current assumptions, this distribution behaves as

$$f\left(\pi_{t+1}|\mathscr{I}_{i,t}^{ref}\right) \sim N\left(\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right], (t-k_i)^{-1}\sigma_\pi^2\right),$$

where

$$\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right] = \frac{\sum_{j=1}^{t-k_i}\mathbb{E}_{i,t-j}^{KF}\left[\pi_{t+1}\right]}{(t-k_i)}.$$

Given these two elements, we define the distribution as

$$f_{i,t}^{\theta}\left(\pi_{t+1}\right) = f\left(\pi_{t+1}|\mathscr{I}_t\right) D_{i,t}^{\theta}\left(\pi_{t+1}\right),$$

where

$$D_{i,t}^{\theta}\left(\pi_{t+1}\right) = \left[\frac{f\left(\pi_{t+1}|\mathscr{I}_t\right)}{f\left(\pi_{t+1}|\mathscr{I}_{i,t}^{ref}\right)^{\frac{1}{t-k_i}}}\right]^{\theta} Z_{i,t},$$

where $Z_{i,t}^{-1} = \int f_{i,t}^{\theta}\left(\pi_{t+1}\right) d\pi_{t+1}$ is a term that ensures that $f_{i,t}^{\theta}\left(\pi_{t+1}\right)$ integrates to one. Therefore, under some algebraic procedure, the probability density function of the distribution can be written as

$$f_{i,t}^{\theta}\left(\pi_{t+1}\right)=\frac{\left[\frac{1}{\sigma_\pi\sqrt{2\pi}}\exp\left\{-\frac{\left(\pi_{t+1}-\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right]\right)^2}{2\sigma_\pi^2}\right\}\right]^{(1+\theta)}Z_{i,t}}{\left[\frac{1}{\sigma_\pi\sqrt{2\pi}}\exp\left\{-\frac{\left(\pi_{t+1}-\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right]\right)^2}{2\sigma_\pi^2}\right\}\right]^{\theta}},$$

And we can make the following approximation

$$f_{i,t}^{\theta}\left(\pi_{t+1}\right)\approx\frac{1}{\sigma_\pi\sqrt{2\pi}}\exp\left\{-\frac{\left(\pi_{t+1}-\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+1}\right]\right)^2}{2\sigma_\pi^2}\right\}Z_{i,t},$$

where

$$\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+1}\right]=\mathbb{E}_{t}^{KF}\left[\pi_{t+1}\right]+\theta\left(\mathbb{E}_{t}^{KF}\left[\pi_{t+1}\right]-\mathbb{E}_{i,t}^{ref}\left[\pi_{t+1}\right]\right).$$

Thus, we conclude that

$$f_{i,t}^{\theta}\left(\pi_{t+1}\right)\sim N\left(\mathbb{E}_{i,t}^{\theta}\left[\pi_{t+1}\right],\sigma_\pi^2\right).$$

# C Monthly inflation as a random walk

Throughout the paper we consider the monthly inflation to be a random walk process. The reason behind is that, with monthly inflation data, we cannot reject the hypothesis of unit root. To further expand on this, in Table A4 we present the results of AR(1) regressions on the monthly US inflation rate from January 1960 to March 2022. Column 1 does not consider a constant while Column 2 does.

Table A4: AR(1) regression for monthly inflation

| | (1) | (2) |
|---|---|---|
| $\pi_{t-1}$ | 0.999*** (0.993 - 1.005) | 0.992*** (0.982 - 1.002) |
| Observations | 747 | 747 |
| R-squared | 0.993 | 0.981 |

**Note**: Table shows the results of AR(1) regressions with the monthly inflation rate in the US from January 1960 to March 2022. Column 1 does not consider a constant while Column 2 does. The current inflation rate measure is based on the monthly YoY percentage variation of the CPI. 95 percent confidence intervals in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

We see that in both specifications the autorregresive coefficient is very close to 1. Furthermore, the value of 1 falls within the 95 percent confidence intervals. Also, the F-test in which the null hypothesis is that the autorregresive coefficient is equal to 1 gives a p-value of 0.66 when we do not consider a constant and a p-value of 0.12 when we consider a constant, such that we cannot reject the null hypothesis in any of the two cases. Lastly, an augmented Dickey-Fuller test on the monthly inflation gives a p-value of 0.52, so that we cannot reject the null hypothesis of a unit root. These findings are in line with those of Pivetta & Reis (2007), who find that there is a very high persistence in the quarterly inflation rate in the US.

Given that we take the inflation rate to be a random walk process, we now turn to the calibration of the Kalman filter of section IV.1. There, we defined the agents assume that inflation behaves as

$$\pi_{t+1} = \pi_t + \varepsilon_t,$$

where $\varepsilon_t$ is a normally independent and identically distributed inflation shock.

Agents also receive a signal $s_t$ given by

$$s_t = \pi_{t+1} + \upsilon_t,$$

where $\upsilon_t$ is a normally independent and identically distributed signal noise.

Furthermore, we assume that the signal is $s_t = \pi_{t-1}^{food}$ where $\pi_{t-1}^{food}$ denotes the inflation in period $t-1$ of the food component of the CPI. By manipulating the equations we get

$$\varepsilon_t = \pi_t - \pi_{t+1},$$

$$\upsilon_t = s_t - \pi_{t+1} = \pi_{t-1}^{food} - \pi_{t+1}.$$

Then, armed with the US monthly inflation and food inflation series from January 1960 to March 2022 we calculate the variances $\sigma_\varepsilon^2$, $\sigma_\upsilon^2$ and the covariance $\sigma_{\varepsilon\upsilon}$.

# D Constant Kalman gain across cohorts

In Section III we argued that cohorts do not react differently to inflation news. Here, we provide further empirical evidence that supports this fact in a structurally-based fashion. We start from the standard Kalman filter specification to obtain

$$\mathbb{E}_{i,t}\left[\pi_{t+1}\right] = \mathbb{E}_{i,t-1}\left[\pi_{t+1}\right] + \kappa\left(s_t - \mathbb{E}_{i,t-1}\left[\pi_{t+1}\right]\right),$$

where $\mathbb{E}_{i,t}\left[\pi_{t+1}\right]$ is the inflation expectation of cohort $i$, $s_t$ is the signal and $\kappa$ is the Kalman gain.

In order to uncover whether cohorts react differently to the signal they are receiving, we estimate $\kappa$ by cohorts as

$$\mathbb{E}^{SCE}_{m,i,t}\left[\pi_{t+1}\right] = \mathbb{E}^{SCE}_{m,i,t-1}\left[\pi_{t+1}\right] + \sum_{i \neq 1941}^{2002} \kappa_i \left(s_t - \mathbb{E}^{SCE}_{m,i,t-1}\left[\pi_{t+1}\right]\right) \times \mathbb{I}_i, \tag{19}$$

where we use the inflation expectations coming from agent $m$ of cohort $i$ in period $t$ from the Survey of Consumer Expectations as the empirical counterpart for the theoretical inflation expectations[34]. $\mathbb{I}_i$ serves as an indicator variable for each cohort, which is defined by the date of birth. If different cohorts react differently to the signal they are receiving, then we should obtain different coefficients across cohorts.

Panel (a) of Figure A9 shows the results. We find that there are no systematically significant differences across cohorts when it comes to the Kalman gain. We interpret this as evidence that supports our fact of cohorts not reacting differently to the inflation signals they receive.

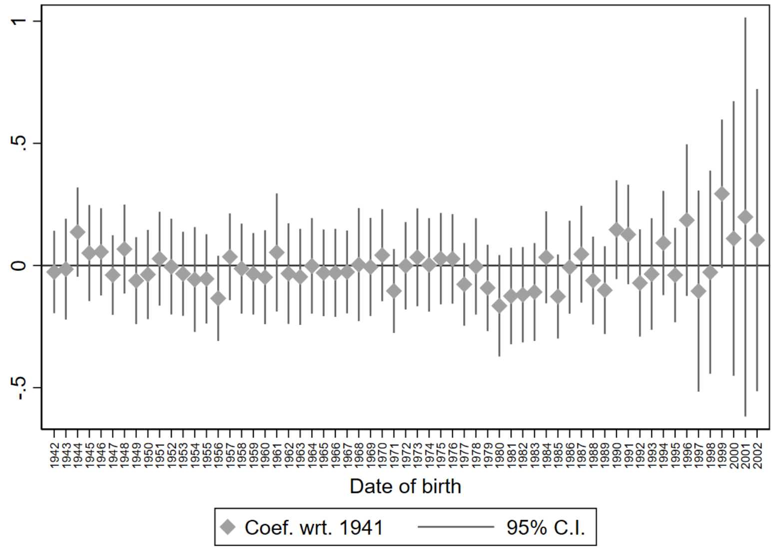
An alternative way of defining cohorts is by the age of the agents at the moment of the survey. Under this definition we estimate

$$\mathbb{E}^{SCE}_{m,i,t}\left[\pi_{t+1}\right] = \mathbb{E}^{SCE}_{m,i,t-1}\left[\pi_{t+1}\right] + \sum_{i \neq 18}^{80} \kappa_i \left(s_t - \mathbb{E}^{SCE}_{m,i,t-1}\left[\pi_{t+1}\right]\right) \times \mathbb{I}_i, \tag{20}$$

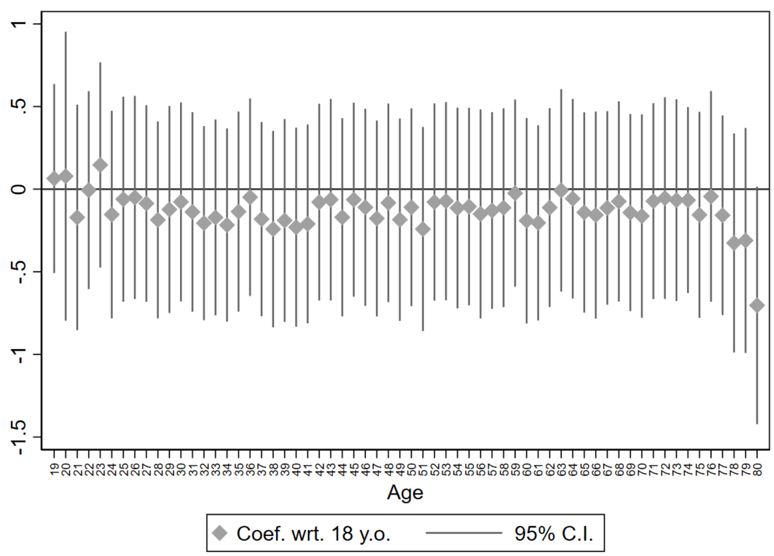
Panel (b) of Figure A9 presents the results. As before, we find there are no systematic differences in the Kalman gain across cohorts when defined by current age.

[34] In order to obtain $\mathbb{E}^{SCE}_{m,i,t-1}\left[\pi_{t+1}\right]$ we assume agents believe inflation behaves as a random walk.

Figure A9: Constant Kalman gain across cohorts




(a) Cohorts by date of birth




(b) Cohorts by age

**Note**: Figure shows the coefficient resulting from estimating Equations 19 and 20. The base cohort in Panel (a) is those born in 1941. The base cohort in Panel (b) is those who were 18 years old at the moment of the survey. We control for period and individual fixed effects.

# E History-based Kalman filter with AR(1) assumption

In this section we repeat the forecasting exercise from Section IV but replacing the random walk assumption with an AR(1) specification. Therefore, agents assume that inflation behaves as

$$\pi_{t+1} = \rho_\pi \pi_t + \varepsilon_t,$$

where the coefficient $\rho_\pi \in [0,1]$ captures the mean-reversion of the inflation variable. Here, we assume the inflation rate has been properly demeaned.

As before, we assume that the signal is given by

$$s_t = \zeta \pi_{t+1} + \upsilon_t.$$

The forecasted value of the inflation variable is

$$\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right] = (1-\zeta K)\,\mathbb{E}_{i,t-1}^{KF}\left[\pi_{t+1}\right] + K s_t,$$

where the difference now lies in the fact that agents use the AR(1) assumption to forecast the inflation rate such that

$$\mathbb{E}_{i,t}^{KF}\left[\pi_{t+h}\right] = \rho_\pi^{h-1}\mathbb{E}_{i,t}^{KF}\left[\pi_{t+1}\right].$$

In this section we assume $\zeta = 1$, $\rho = 0.99$, $\sigma_\varepsilon = 0.15$, $\sigma_\nu = 4.09$ and $\sigma_{\varepsilon\nu} = -0.06$.[35] This gives $K = 0.175$.

For the different cohorts Panel (a) of Figure A10 presents the standard Kalman filter forecast, while Panel (b) of Figure A10 presents the reference.[36]

Table A5 presents the result of the parameter estimation. In this case, $\theta = -0.526$. Armed with this coefficient, Panel (c) of Figure A10 shows the heterogeneous forecasts across cohorts.

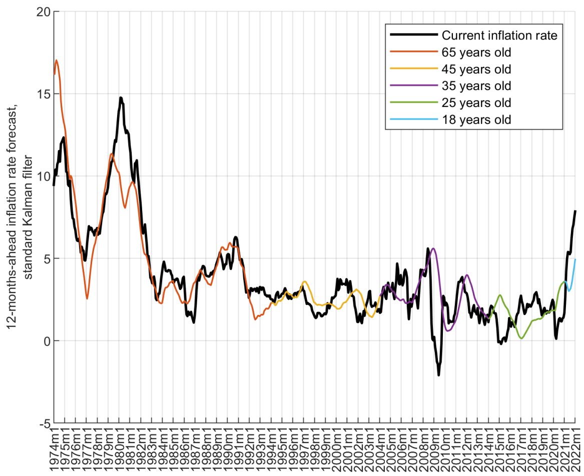
Finally, Figure A11 presents the comparison of the forecasts with the AR(1) assumption and the observed forecasts in the data. We find that, as with the random walk process, this version of the

[35] We calibrate the variances and covariance according to Appendix C.

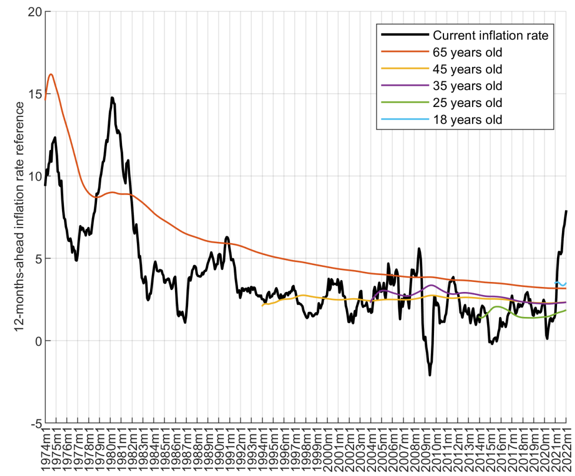
[36] We return the mean to the data before plotting the graphs, where the long-run mean of the inflation rate is 2 percent.

forecast based on an AR(1) assumption provides a good fit to the data.

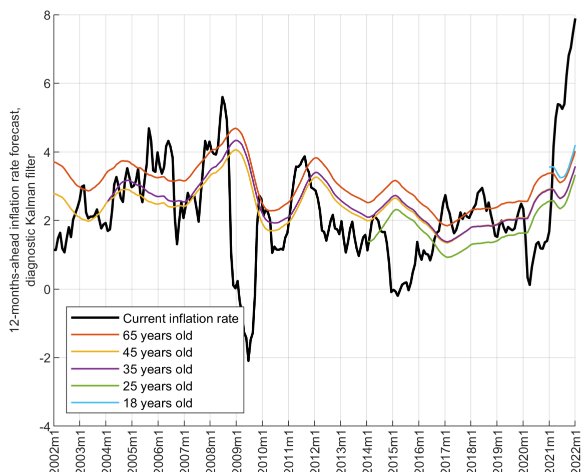
Figure A10: History-based Kalman-filter-based inflation forecasts by cohort, AR(1)




(a) Standard Kalman-filter-based inflation forecasts




(b) Inflation rate reference




(c) History-based Kalman-filter-based inflation forecasts

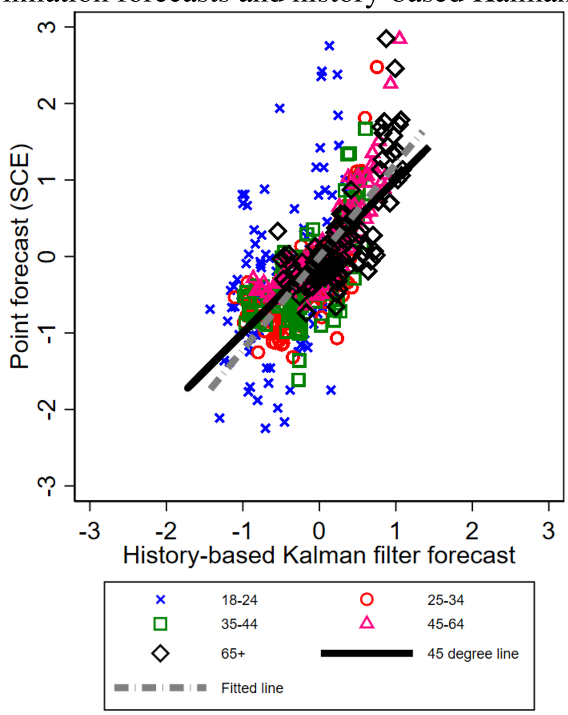
**Note**: Panel (a) shows the Kalman filter forecast for the common component for selected cohorts, differentiated by their age in 2021. Panel (b) shows the references for selected cohorts obtained according to the Kalman filter and given the history of inflation experienced by the corresponding age group. Panel (c) shows forecasts for selected cohorts according to the Kalman-filter-augmented expectations and considering the estimate for $\theta$ from Column 1 of Table 2. Selected cohorts are differentiated by their age in 2021. We further assume that each cohort starts forecasting when they become 18 years old. The current inflation rate measure is based on the monthly YoY percentage variation of the CPI.

Table A5: Parameter estimation, AR(1)

| | (1) |
|---|---|
| $\mathbb{E}_{i,t}^{ref}\left[\pi_{t+12}\right]$ | 0.526*** |
| | (0.047) |
| Time FE | Yes |
| Observations | 101,262 |
| R-squared | 0.092 |

**Note**: Table shows results of Regression 6. $\mathbb{E}_{i,t}^{ref}\left[\pi_{t+12}\right]$ is the reference constructed for a respondent of age $i$ as explained in the main text. Column 1 has only a time fixed effect as an additional control. Robust standard errors in parentheses. Standard errors clustered by age. Dependent variable trimmed at 10 percent and 90 percent in each period. *** p<0.01, ** p<0.05, * p<0.1.

Figure A11: Observed inflation forecasts and history-based Kalman filter forecasts, AR(1)




**Note**: Figure shows binned scatterplot across history-based Kalman filter forecasts (x-axis) and point forecast inflation expectations according to the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York (y-axis). Variables demeaned by the intercept. Data go from June 2013 to December 2021. SCE variable trimmed at 10 percent and 90 percent in each period.

# F History-based Kalman filter with alternative signals

Throughout the paper we use the lagged inflation rate of the food component of the CPI as a signal of the non-observed aggregate inflation variable. This is based on previous papers study how consumers form their inflation expectations based on shopping experiences and food prices (D'Acunto et al., 2021; D'Acunto & Weber, 2022). In this section we consider two alternative components of the CPI as the signals consumers use: food at home and dairy.

Table A6 presents the estimation of the parameter according to each signal. In all cases we find a positive weight on the reference. Moreover, the parameter is close to the baseline from Table 2.

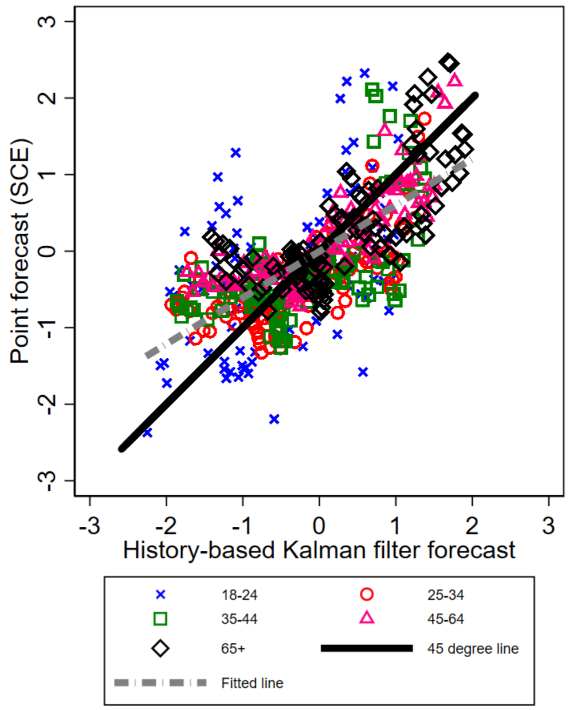
Figure A12 shows how the history-based Kalman filter forecasts with alternative signals fare against the observed inflation forecasts. Using the food at home and dairy CPI components as a signal provides a decent fit to the observed data.

Table A6: Main parameter estimation, alternative signals

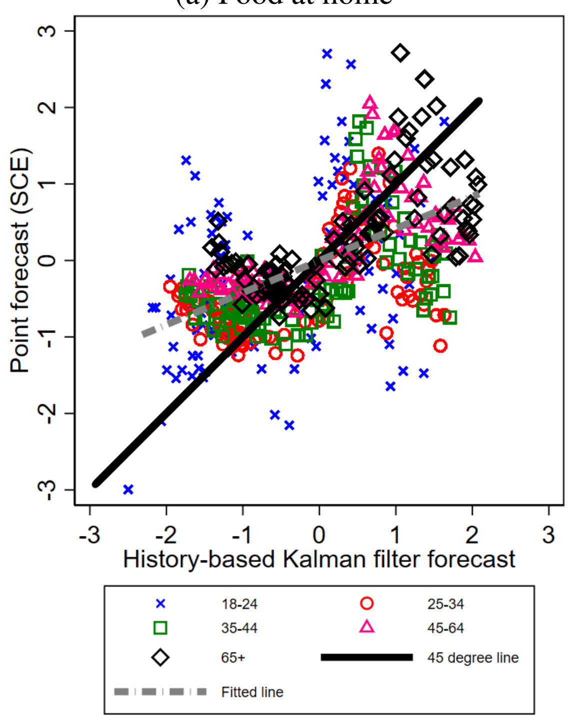
| | Food at home | Dairy |
|---|---|---|
| $\mathbb{E}_{i,t}^{ref}[\pi_{t+12}]$ | 0.307*** | 0.334*** |
| | (0.027) | (0.029) |
| Observations | 101,262 | 101,262 |
| R-squared | 0.092 | 0.092 |

**Note**: Table shows results of Regression 6 using alternative CPI components as the signal. $\mathbb{E}_{i,t}^{ref}[\pi_{t+12}]$ is the reference constructed for a respondent of age $i$ as explained in the main text. Columns have only a time fixed effect as an additional control. Robust standard errors in parentheses. Standard errors clustered by age. Dependent variable trimmed at 10 percent and 90 percent in each period. *** p<0.01, ** p<0.05, * p<0.1.

Figure A12: Observed inflation forecasts and history-based Kalman filter forecasts, alternative signals




(a) Food at home




(b) Dairy

**Note**: Figure shows binned scatterplot across history-based Kalman filter forecasts (x-axis) and point forecast inflation expectations according to the Survey of Consumer Expectations (SCE) of the Federal Reserve Bank of New York (y-axis) using alternative CPI components as the signal. Variables demeaned by the intercept. Data go from June 2013 to December 2021. SCE variable trimmed at 10 percent and 90 percent in each period.

# G External validity: European data

We check the external validity of our results using data from the Consumer Expectations Survey (CES) of the European Central Bank. It contains monthly data between April 2020 and September 2022 for six countries: Belgium, France, Germany, Italy, the Netherlands, and Spain.[37]

We see that in Europe, lifetime experiences with the inflation rate are also heterogeneous across cohorts, as we show in Figure A14. Moreover, we see that by 2020 the youngest cohorts had not been exposed to high inflation rates, but this changes after the high inflation rate episode of 2021 and 2022. After this, the youngest cohorts are the ones that show the highest lifetime average for the inflation rate, even larger than that of the people who experienced the high inflation rates of the 80s.

In Figure A15 we relate the two previous facts and find that in Europe, similar to the US, the larger the inflation rate individuals have experienced in their lifetimes, the higher their inflation expectations.

Table A7 shows that in Europe, as happened in the US, after controlling for the average lifetime inflation rate, younger generations do not react more strongly to inflation news than older cohorts.[38]

We now turn to the history-based Kalman filter of Section IV.1. Table A8 shows the parameters that go into the Kalman filter calibration after using European inflation rate and food inflation rate. Then, we estimate the parameter according to Equation 6.[39] In Table A9 in our baseline specification of Column 1 we find $\theta^{eur} = -0.156$. With this parameter, in Figure A16 we plot inflation expectations according to our history-based Kalman filter, across cohorts and in each of the six countries in our sample. We find that the oldest cohorts have the highest inflation expectations before 2021. Then, after 2021 the youngest cohorts start catching up with the oldest ones and even

[37] There is a relevant difference between the data sets of US and Europe. In the former we have the exact age of the respondents. In the latter we do not have detailed information on the age of the respondents, as they are classified in 4 age groups: 18-34, 35-49, 50-70 and 71+.

[38] We confirm the finding with a F-test where the null hypothesis is that all of the interactions are jointly equal to zero. The test gives a p-value of 0.35, so we cannot reject the null hypothesis.

[39] Because we do not know the exact age of the respondents, we do not know which are the exact lifetime average inflation rates they have experienced. Therefore, for this estimation, we assume that every agent in cohort 18-34 has the lifetime average inflation rate of a 25-year-old, every agent in cohort 35-49 has the lifetime average inflation rate of a 35-year-old, every agent in cohort 50-70 has the lifetime average inflation rate of a 50-year-old and every agent in cohort 71+ has the lifetime average inflation rate of a 71-year-old. On the signals used, because the series on the inflation rate of the food component of the CPI have varying starting dates in the different countries, we replace the missing values with the observed inflation rate in order to make the starting dates of all countries uniform.

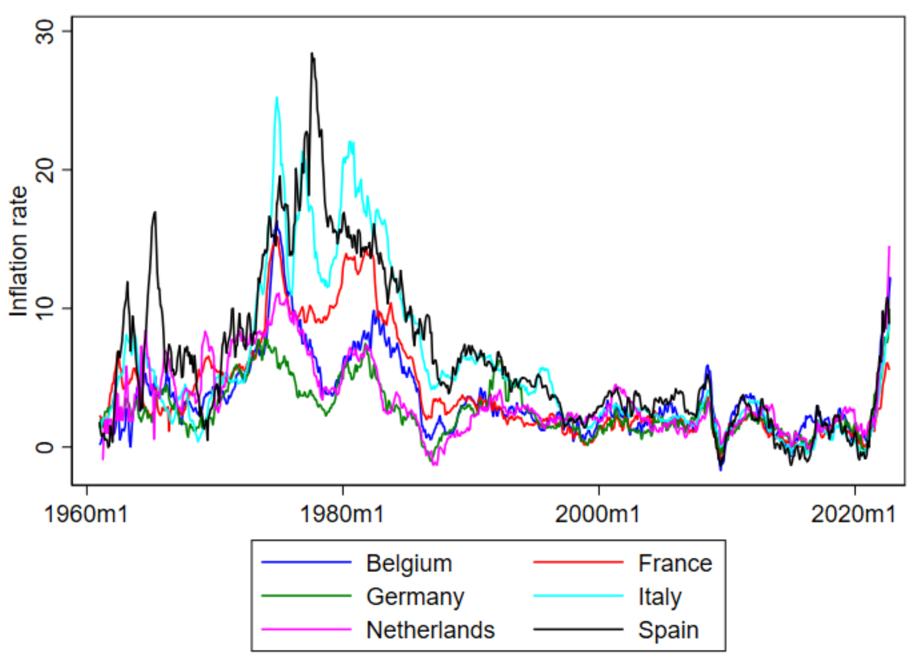
surpass them in some countries.

Table A9 also shows additional specifications of the estimation of Equation 6. We find that after controlling for cohort and country fixed effects, we still find that agents weight positively their references when forming their expectations. These additional specifications also tell us that the heterogeneity in expectations across cohorts is not due to people of different ages or from different countries facing different consumption bundles or having different preferences, but to the proposed anchoring-to-the-past mechanism. Thus, it is past experiences that define expectations, not the age or the geographic location per se.[40]

Lastly, in Figure A17 we compare the inflation expectations generated by our history-based Kalman filter to the survey data. We see that we have a decent fit to the data.

We conclude that our findings from the main text are also valid for Europe. We find evidence that supports the claim that (i) inflation expectations are also heterogeneous in Europe and (ii) can also be modeled by a history-based Kalman filter with positive weight on their historical reference.

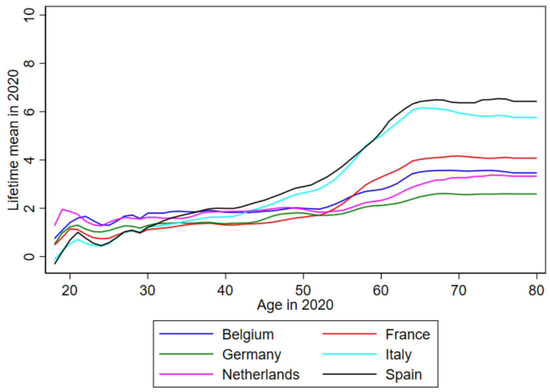
Figure A13: Inflation rate, Europe



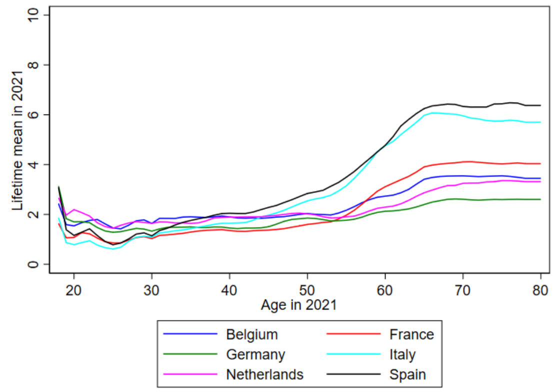

**Note**: The current inflation rate measure is based on the monthly YoY percentage variation of the CPI.
**Source**: FRED.

[40]See Hajdini et al. (2022a) for a further discussion on this.

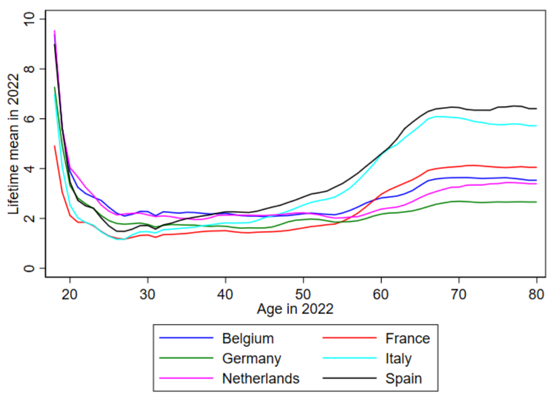
Figure A14: Lifetime average inflation rate among respondents, Europe




(a) 2020



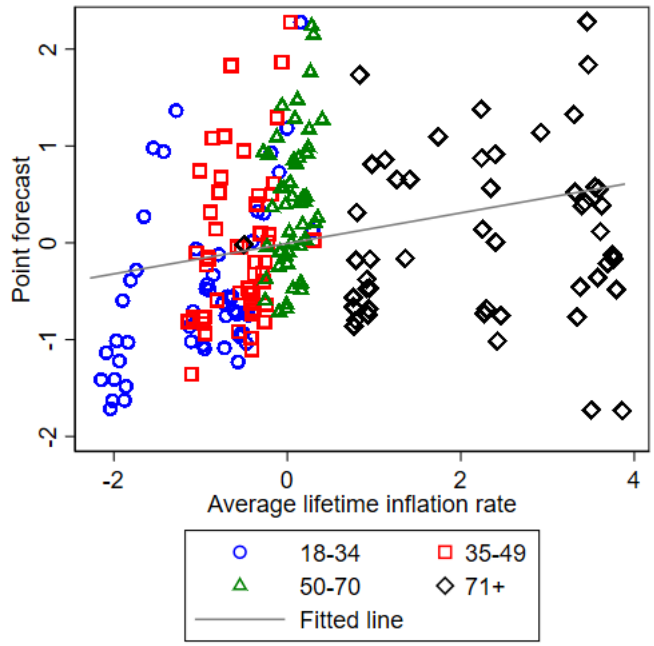

(b) 2021




(c) 2022

**Note**: Figure shows the mean of the monthly YoY inflation rate that people of the age shown in 2020, 2021, and 2022 have experienced in their lifetimes, starting when they were age 18. The current inflation rate measure is based on the monthly YoY percentage variation of the CPI.
**Source**: FRED.

Figure A15: Inflation point forecast and average lifetime inflation rate, Europe




**Note**: Figure shows binned scatterplot across lifetime average inflation rate bins. Variables residualized by respondent gender and commuting zone. Data go from April 2020 to September 2022. Ages correspond to the interviewee's age at the time of the survey. The average lifetime inflation rate measure is based on the monthly YoY percentage variation of the CPI.
**Source**: Consumer Expectations Survey.

Table A7: Effects of current and experienced inflation rates on inflation expectations

| Dep. var.: Inflation expectations | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Average lifetime inflation rate | 0.276* | 0.244** | 0.301* | 0.252* |
| | (0.132) | (0.100) | (0.149) | (0.129) |
| Current inflation | 0.351*** | 0.275*** | | |
| | (0.033) | (0.057) | | |
| Cohort 35-49 | | 0.155 | | |
| | | (0.090) | | |
| Cohort 50-70 | | 0.203* | | |
| | | (0.112) | | |
| Cohort 71+ | | -0.401 | | |
| | | (0.348) | | |
| Current inflation $\times$ 35-49 | | 0.074 | | |
| | | (0.074) | | |
| Current inflation $\times$ 50-70 | | 0.122 | | |
| | | (0.069) | | |
| Current inflation $\times$ 71+ | | 0.124 | | |
| | | (0.083) | | |
| Time FE | No | No | Yes | Yes |
| Controls | No | No | No | Yes |
| Observations | 294,232 | 294,232 | 294,232 | 294,232 |
| R-squared | 0.128 | 0.135 | 0.152 | 0.164 |

**Note**: Table shows regressions where the dependent variable is inflation expectations according to the Consumer Expectations Survey (CES) of the European Central Bank. Column 1 shows controls for the average lifetime inflation of respondents of a given age at each period in time and the last inflation measure. Column 2 follows Column 1 but adds cohort fixed effects and the interaction of those cohort fixed effects with the current inflation. Column 3 follows Column 1 but adds time fixed effects and, hence, omits the current inflation variable. Column 4 follows Column 1 but adds time fixed effects and demographic controls. The demographic controls are income, gender, educational level, and country. The current inflation rate measure is based on the monthly YoY percentage variation of the CPI. Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1. Standard errors clustered by age. The dependent variable is trimmed, dropping the lower and upper 10 percent of answers in each period.

Table A8: History-based Kalman filter parameters, Europe

| | $\sigma_\varepsilon^2$ | $\sigma_\upsilon^2$ | $\sigma_{\varepsilon\upsilon}$ | $K$ |
|---|---|---|---|---|
| Belgium | 0.16 | 2.51 | -0.28 | 0.24 |
| France | 0.09 | 2.63 | -0.14 | 0.17 |
| Germany | 0.12 | 3.13 | -0.15 | 0.18 |
| Italy | 0.23 | 3.38 | -0.38 | 0.24 |
| Netherlands | 0.13 | 3.54 | -0.18 | 0.18 |
| Spain | 0.45 | 5.83 | -0.65 | 0.26 |

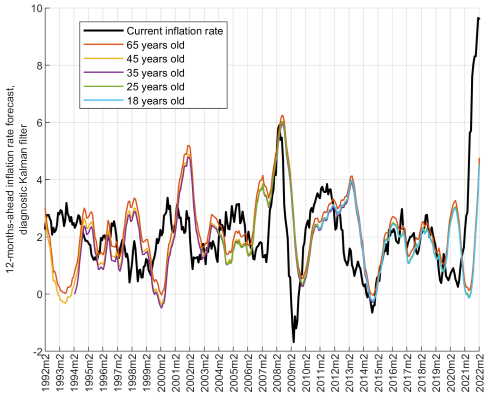
**Note**: We obtain this calibration for each country following the steps outlined in Appendix C. The data for these calculations goes from January 1971 to October 2022.

Table A9: Parameter estimation, Europe

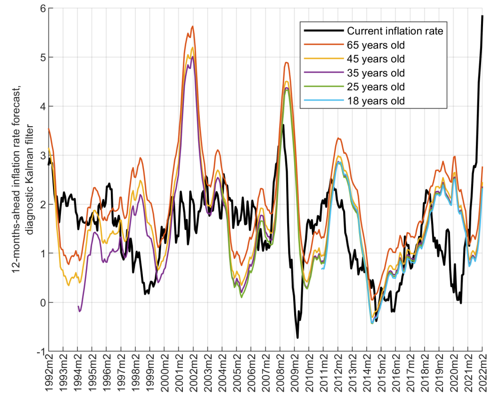
| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $\mathbb{E}_{i,t}^{ref}[\pi_{t+12}]$ | 0.156*** | 0.208*** | 0.094*** | 0.060* |
| | (0.025) | (0.044) | (0.019) | (0.030) |
| Time FE | Yes | Yes | Yes | Yes |
| Controls | No | Cohort | Country | Cohort, country |
| Observations | 271,311 | 271,311 | 271,311 | 271,311 |
| R-squared | 0.122 | 0.130 | 0.132 | 0.140 |

**Note**: Table shows results of Regression 6, but changing the dependent variable for inflation expectations according to the Consumer Expectations Survey (CES) of the European Central Bank. The independent variable $\mathbb{E}_{i,t}^{ref}[\pi_{t+12}]$ is the reference constructed for a respondent of age $i$ as explained in the main text. Column 1 considers a time fixed effect as a control. Column 2 has time and cohort fixed effects. Column 3 has time and country fixed effects. Column 4 has time, cohort, and country fixed effects. Standard errors clustered by age in parentheses. Dependent variable trimmed at 10 percent and 90 percent in each period. We use population weights. *** p<0.01, ** p<0.05, * p<0.1.

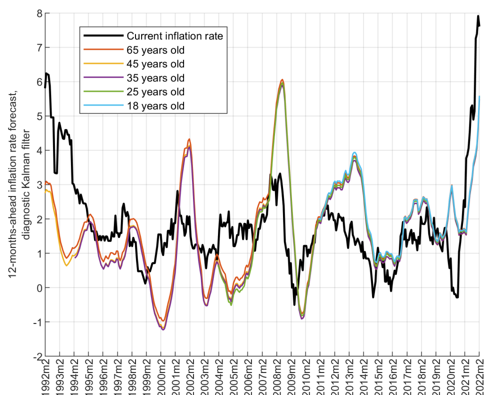
Figure A16: History-based Kalman-filter-based inflation forecasts by cohort, Europe



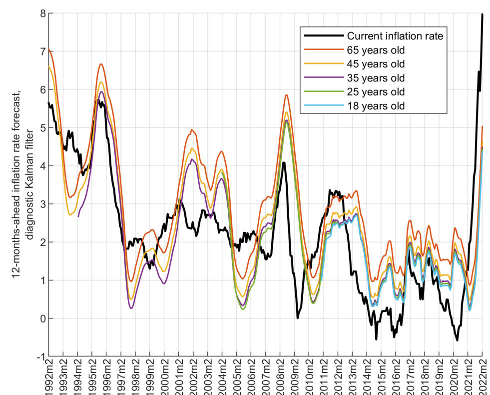

(a) Belgium



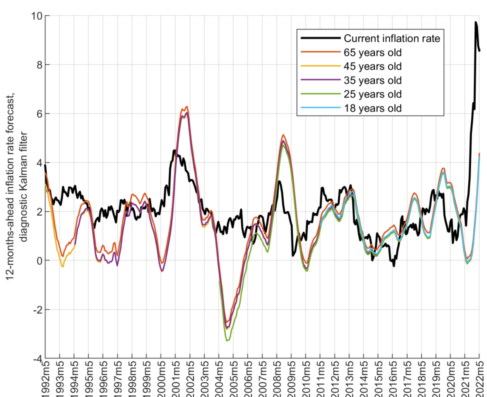

(b) France



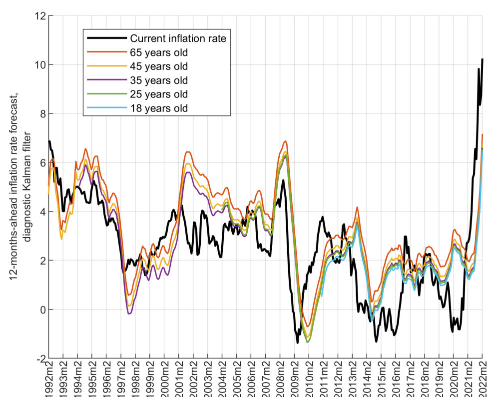

(c) Germany




(d) Italy



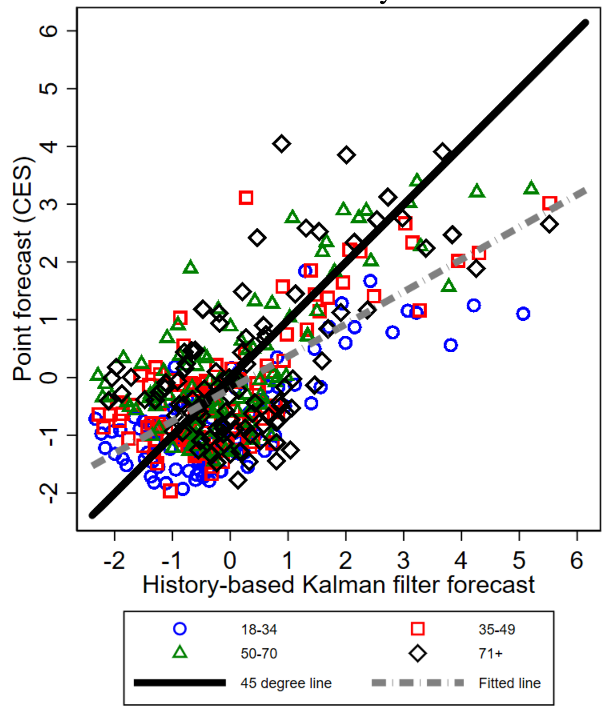

(e) Netherlands




(f) Spain

**Note**: Figure shows forecasts for selected cohorts according to the Kalman-filter-augmented expectations and considering the estimate for $\theta$ from Column 1 of Table A9. Selected cohorts differentiated by their age in 2021. We further assume that each cohort starts forecasting when they become 18 years old. The current inflation rate measure is based on the monthly YoY percentage variation of the CPI.

Figure A17: Observed inflation forecasts and history-based Kalman filter forecasts, Europe




**Note**: Figure shows binned scatterplot across history-based Kalman filter forecasts (x-axis) and point forecasts of inflation expectations according to the Consumer Expectations Survey (CES) of the European Central Bank (y-axis). Variables demeaned by the intercept. Data go from April 2020 to September 2022. SCE variable trimmed at 10 percent and 90 percent in each period.

# H Derivations for firm block

## H.1 Final good producer

The final good producer operates in a perfectly competitive market. It produces the final good $Y_t$ using a CES basket composed by a continuum of intermediate goods $Y_t(j)$ with $j \in [0,1]$. Therefore, the optimization problem of the firm is

$$\max_{\{Y_t(j)\}_{j\in[0,1]}} P_t Y_t - \int_{j\in[0,1]} P_t(j) Y_t(j)\, dj,$$

subject to

$$Y_t = \left[ \int_{j\in[0,1]} Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $P_t(j)$ is the price of intermediate good $j$, and $\varepsilon > 1$ is the elasticity of substitution in the CES basket. As a solution, we obtain the corresponding downward-sloping demand curve $\forall$ $j \in [0,1]$

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t,$$

where the price of this production basket is given by

$$P_t = \left[ \int_{j\in[0,1]} P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}},$$

## H.2 Intermediate good producers

There is a continuum of monopolistically competitive firms indexed by $j \in [0,1]$. Each firm $j \in [0,1]$ produces the intermediate good $Y_t(j)$ with a production function given by

$$Y_t(j) = A_t L_t(j),$$

where $A_t$ is a process that represents technology, and $L_t(j)$ is the labor employed by firm $j$. The intermediate good producer will pay a nominal wage $w_t$ to workers.

We assume that intermediate good producers face price rigidities à la Calvo, where the probability of price change corresponds to $(1-\phi) \in (0,1)$. Therefore, a reoptimizing intermediate good producer $j \in [0,1]$ solves

$$\max_{\{P_t(j)\}} \mathbb{E}_t \left[ \sum_{k \geq t} \phi^{k-t} Q_{t,k} \left( P_t(j) Y_{k|t}(j) - \Psi_k \left( Y_{k|t}(j) \right) \right) \right],$$

where $\Psi_t(.)$ is a well-defined cost function. Here, $Q_{t,k} = \beta^{k-t} (C_k/C_t)^{-\sigma} (P_t/P_k)$ with $k \geq t$ is the household's stochastic discount factor used by firms. We assume that intermediate goods producers use the stochastic discount factor associated with the aggregate consumption level. This optimization is subject to the following sequence of demand for $k \geq t$:

$$Y_{k|t}(j) = \left( \frac{P_t(j)}{P_k} \right)^{-\varepsilon} Y_k.$$

The first-order condition of this problem leads to

$$Y_{t|t}(j) \left[ 1 - \mu MC_{t|t}(j) \left( \frac{P_t}{P_t(j)} \right) \right] + \mathbb{E}_t \left[ \sum_{k>t} \phi^{k-t} Q_{t,k} Y_{k|t}(j) \left[ 1 - \mu MC_{k|t}(j) \left( \frac{P_k}{P_k(j)} \right) \right] \right] = 0,$$

where $MC_{k|t}(j) = \Psi_k' \left( Y_{k|t}(j) \right) / P_k$ denotes real marginal costs for $k \geq t$, and $\mu = \varepsilon/(\varepsilon - 1) > 1$ is the desired frictionless mark-up. Under the assumption of symmetry across all firms, the solution implies $P_t(j) = P_t \; \forall \; j \in [0,1]$. Therefore, the previous first-order condition reduces to

$$Y_{t|t}(j) \left( 1 - \mu MC_{t|t}(j) \right) + \mathbb{E}_t \left[ \sum_{k>t} \phi^{k-t} Q_{t,k} Y_{k|t}(j) \left( 1 - \mu MC_{k|t}(j) \right) \right] = 0.$$

Regarding the cost structure of this model, i.e. the $\Psi_t(.)$ function, we assume that the only cost of the firms are labor costs

$$\Psi_k \left( Y_{k|t}(j) \right) = \frac{W_k Y_{k|t}(j)}{A_k},$$

which implies $MC_{k|t}(j) = W_k / (A_k P_k) = MC_k \; \forall \; j \in [0,1]$ and

$$\mathscr{M}_t = P_t MC_t \quad = \quad \frac{W_t}{A_t}.$$

Using the aggregate market clearing condition in the market of goods, i.e. $Y_t = C_t\ \forall t$, the optimal price $P_t^*$ must satisfy:

$$P_t^* = \mu \frac{\mathbb{E}_t\left[\sum_{k\geq t}\phi^{k-t}Q_{t,k}Y_{k|t}(j)\,\mathscr{M}_k\right]}{\mathbb{E}_t\left[\sum_{k\geq t}\phi^{k-t}Q_{t,k}Y_{k|t}(j)\right]} \quad = \quad \mu \frac{\mathbb{E}_t\left[\sum_{k\geq t}(\phi\beta)^{k-t}P_k^{\varepsilon}C_k^{1-\sigma}MC_k\right]}{\mathbb{E}_t\left[\sum_{k\geq t}(\phi\beta)^{k-t}P_k^{\varepsilon-1}C_k^{1-\sigma}\right]}.$$

A log-linearization of the previous equation leads to the following condition for the optimal price

$$p_t^* = \mu + (1-\phi\beta)\,\mathbb{E}_t\left[\sum_{k\geq t}(\phi\beta)^{k-t}(p_k + mc_k)\right].$$

Finally, and following the standard procedure in the literature, we can derive the New Keynesian Phillips curve. Under our framework, this equations corresponds to

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \frac{(1-\phi)(1-\phi\beta)}{\phi}(\sigma+\eta)\,y_t.$$

# I Heterogeneous cohorts

In the baseline exercises we assumed a history-based Kalman filter operator where history varies by cohort, as in Equation 3. In this section we analyze a variation of such operator, where we assume that history is fixed across all cohorts. We define this alternative history-based Kalman filter operator as

$$E_t^{\theta,alt}[X_{t+h}] = E_t^{KF}[X_{t+h}] + \theta\left(E_t^{KF}[X_{t+h}] - \sum_{j=1}^{J}\frac{E_{t-j}^{KF}[X_{t+h}]}{J}\right),$$

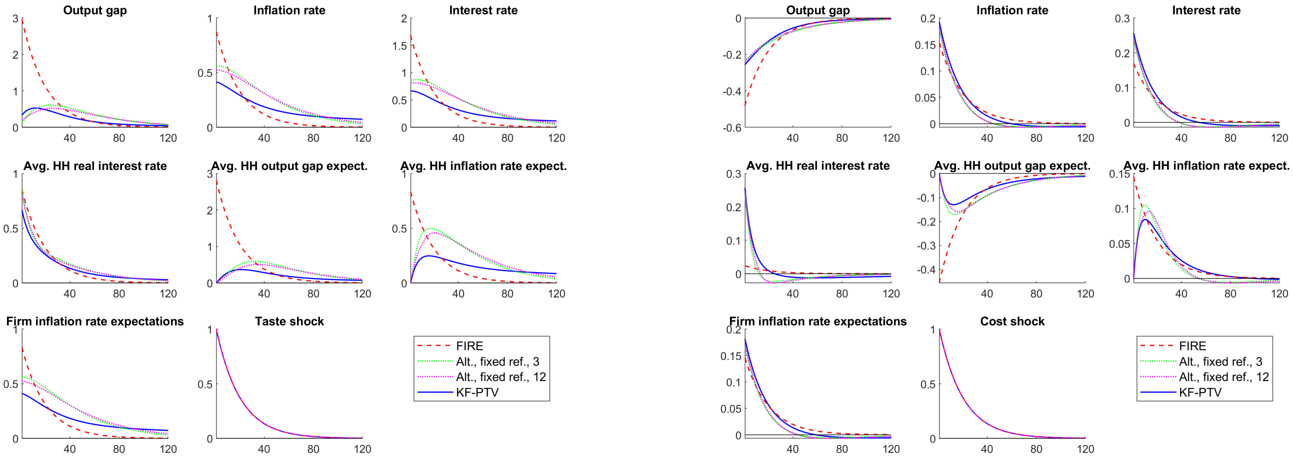
where agents remember what occurred in the last $J$ periods. As in the baseline case, we set $\theta = -0.317$.

Figure A18 shows the impulse response functions to a cost shock and a taste shock according to our model of Section V. Besides the responses coming from the FIRE and history-based Kalman filter cases with varying history across cohorts, we also consider the two cases for the alternative history-based Kalman filter operator: fixing the history of all cohorts to the last 3 periods and fixing the reference of all cohorts to the last 12 periods.

We see that the three cases follow the same pattern when compared to the FIRE case. However, the expectations under the fixed reference cases show a stronger reaction to the shocks. This is because the history span in these two alternative fixed reference cases is shorter than in the full-fledged history-based case. While the latter remembers and is pegged to the steady state for longer, the former cases do not.

It could be argued that if we choose a sufficiently long history with the alternative fixed-reference Kalman filter operator, then we could closely replicate the results coming from the full-fledged history-based case with different cohorts. However, we prefer the full-fledged history-based case with different cohorts as it picks up the richness of the data and introduces it in the model: there are different cohorts living at the same time, each has had different life experiences and each has different beliefs.

Figure A18: Impulse response functions, comparison with alternative history-based Kalman filter operator




(a) Taste shock

(b) Cost shock

**Note**: Figure shows impulse response functions for a selected group of variables after the mentioned shocks. The red dashed line shows the results for the case of the full information rational expectations model (FIRE), the green and magenta dotted lines show the results of history-based Kalman filter model with fixed reference and the solid blue line shows the history-based Kalman filter model where the reference varies by cohort. Horizontal axis denotes months after the shock.